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Codes correcting and simultaneously detecting solid burst errors

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ABSTRAKSI

Mendeteksi dan mengoreksi kesalahan adalah salah satu tugas utama dalam teori pengkodean. Batas penting dalam hal kesalahan mendeteksi dan mengoreksi kemampuan kode. Kesalahan ledakan padat adalah umum di beberapa saluran komunikasi. Makalah ini memperoleh batas bawah dan batas atas pada jumlah dijit paritas-cek yang diperlukan untuk kode linier mampu mengoreksi kesalahan ledakan padat dari panjang b atau kurang dan sekaligus mendeteksi kesalahan ledakan solid dari panjang s (> b) atau kurang. Ilustrasi kode seperti itu juga disediakan.

Kata Kunci: cek paritas matriks, sindroma, kesalahan ledakan padat, array standar

ABSTRACT

Detecting and correcting errors is one of the main tasks in coding theory. The bounds are important in terms of error-detecting and -correcting capabilities of the codes. Solid Burst error is common in several communication channels. This paper obtains lower and upper bounds on the number of parity-check digits required for linear codes capable of correcting any solid burst error of length b or less and simultaneously detecting any solid burst error of length s(>b) or less. Illustration of such a code is also provided.

Keyword: parity check matrix, syndromes, solid burst errors, standard array

1. INTRODUCTION

One of the most important investigations in coding theory is the detection and correction of errors. In this direction, numerous works were done by various mathematicians. Many important/famous codes (like Hamming codes, Golay codes, BCH codes) were developed to combat errors which occurred during communication and these codes were found applications in numerous areas of practical interest. There is a long history towards the growth of the subject. One of the areas of practical importance in which a parallel growth of the subject took place is that of burst error detecting and correcting codes. It is due to the fact that burst errors occur more frequently than random errors in many communication channels. A burst of length *b* may be defined as follows:

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Definition 1: A burst of length b is a vector whose only non-zero components are among some b consecutive components, the first and the last of which is non zero.

If the components inside the b consecutive components are all non zero i.e., all the digits among the b components are in error, such type of burst is known as solid burst. There are channels (viz. semiconductor memory data [10], supercomputer storage system [2]) where solid burst occurs. A solid burst may be defined as follows:

Definition 2: A solid burst of length b is a vector with non zero entries in some b consecutive positions and zero elsewhere.

Schillinger [16] developed codes that correct solid burst error. Shiva and Cheng [18] also produced a paper for correcting multiple solid burst error of length b in binary code with a very simple decoding scheme. For more works on solid burst error, one may refer Bossen [3], Sharma and Dass [17], Etzion [8], Argyrides et al. [1], Das [4, 5, 6], etc.

It is important to know the ultimate capabilities and limitation of error correcting codes. This information, along with the knowledge of what is practically achievable, indicates which problems are virtually solved and which needs further work. Hamming [9] was the first concerned with both code constructions and bounds. The bounds on the number of parity check digits are important from the point of efficiency of a code. The lesser of parity check symbols in a code, the more is the rate of information of the code.

Lower and upper bounds are known for a code capable of correcting/detecting solid burst error [4]. In order to achieve the ability of correcting errors, a large number of check digits is required. The extra check digits could be saved if only error detection, but no error correction after a level, is required. Therefore, a study of simultaneous correction and detection of error is required. In this regard for example, some authors (see, e.g., [7, 11, 14]) studied codes capable of correcting and simultaneously detecting certain type of errors. In view of this, the paper obtains bounds on parity check digits of a code capable of correcting solid burst of length b or less and simultaneously detecting any solid burst of length $s(>b)$ or less.

The paper is organized as follows:
Section 1 i.e., the Introduction gives brief view of the importance of bounds on parity check digits of a code and the requirement for consideration of solid burst errors. Section 2 gives a lower bound on the number of parity check digits of a linear code that corrects any solid burst of length b or less and simultaneously detects any solid burst of length $s(>b)$ or less. Section 3 gives an upper bound on the number of parity check digits for such a linear code. An illustration is also given in section 4. Section 5 gives the discussion and conclusion.

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Note that a (n, k) linear code that is capable of correcting any solid burst of length b or less and simultaneously detecting any solid burst of length $s(>b)$ or less requires the following conditions to be satisfied:

(i) The syndrome resulting from the occurrence of a solid burst error of length b or less must be non-zero and distinct from the syndromes resulting from any other solid burst errors of length b or less.

(ii) The syndrome resulting from the occurrence of solid burst error of length l ($s \geq l > b$) must be nonzero and different from those syndromes resulting from solid burst error of length b or less, (but may be same among themselves).

In what follows a linear code will be considered as a subspace of the space of all n-tuples over the finite field of q elements GF(q). The distance between two vectors shall be considered in the Hamming sense.

2. A LOWER BOUND

We consider linear codes over GF(q) that are capable of correcting any solid burst of length b or less and simultaneously detecting any solid burst of length $s(>b)$ or less. Firstly, a lower bound over the number of parity-check digits required for such a code is obtained. The proof is based on the technique used in theorem 4.16, Peterson and Weldon [13].

Theorem 1. Any (n, k) linear code over GF(q) that corrects any solid burst of length b or less and simultaneously detects any solid burst of length $s(>b)$ or less must have at least $loq_q\left\{2+\sum_{i=1}^{b}(n-i+1)(q-1)^i\right\}$ parity-check digits.

Proof. Let there be an (n, k) linear code vector over GF(q) that corrects all solid bursts of length b or less and simultaneously detects any solid bursts of length $s(>b)$ or less. The maximum number of distinct syndromes available using n-k check bits is q^{n-k}. The proof proceeds by first counting the number of syndromes that are required to be distinct by condition (i) and different syndromes (from those obtained from (i)) by condition (ii) and then setting this number less than or equal to q^{n-k}. Since the code is capable of correcting all errors which are in form of solid burst of length b or less, any syndrome produced by a solid burst of length b or less must be different from any such syndrome likewise resulting from solid burst of length b or less by condition (i).

Also by condition (ii), syndromes produced by solid burst of length $s(>b)$ or less must be nonzero and different from those obtained by condition (i). The number of syndromes, obtained by condition (i) and (ii), is atleast

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$$1+\sum_{i=1}^{b}(n-i+1)(q-1)^{i}\ .\ \text{(refer [4])}$$

Thus the total number of such syndromes including the vector of all zeros are atleast

$$2+\sum_{i=1}^{b}(n-i+1)(q-1)^{i}\ .$$

Therefore we must have

$$q^{n-k}\geq 2+\sum_{i=1}^{b}(n-i+1)(q-1)^{i}\ . \qquad (1)$$

$$\Rightarrow \qquad n-k\ \geq\ log_{q}\left\{2+\sum_{i=1}^{b}(n-i+1)(q-1)^{i}\right\}.$$

3. AN UPPER BOUND

In the following theorem, an upper bound on the number of check digits required for the construction of a linear code considered in theorem 1 is provided. This bound assures the existence of such a linear code. The proof is based on the well known technique used in Varshomov-Gilbert Sacks bound by constructing a parity check matrix for such a code (refer Sacks [15], also theorem 4.7 Peterson and Weldon [13]).

Theorem 2. There shall always exist an (n, k) linear code over GF(q) that corrects solid burst of length b or less and simultaneously detects any solid burst of length s($>b$) or less provided that

$$q^{n-k}>1+\sum_{i=1}^{b}\sum_{l=1}^{b}(n-l-i+1)(q-1)^{i+l-1}+\sum_{i=b}^{s-1}(q-1)^{i}\ . \qquad (2)$$

Proof. The existence of such a code will be proved by constructing an $(n-k)\times n$ parity check matrix H for the desired code as follows.

Select any non zero (n-k)-tuple as the first column h_1 of the matrix H. After having selected the first j-1 columns $h_1, h_2, \ldots, h_{j-1}$ appropriately, we lay down the condition to add j^{th} column h_j as follows:

According to the condition (i), h_j should not be a linear sum of immediately preceding up to b-1 consecutive columns $h_{j-1}, h_{j-2}, \ldots, h_{j-b+1}$, together with any b

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or fewer consecutive columns from amongst the first j-b columns $h_1, h_2, \ldots, h_{j-b}$ i.e.,

$$h_j \neq (u_{j-1}h_{j-1} + u_{j-2}h_{j-2} + \ldots\ldots + u_{j-a+1}h_{j-a+1} + u_{j-a}h_{j-a})$$

$$+ (v_i h_i + v_{i+1}h_{i+1} + \ldots\ldots + v_{i+a'-2}h_{i+a'-2} + v_{i+a'-1}h_{i+a'-1})$$

where $u_i, v_i \in GF(q)$ are non zero coefficients; $a \leq b$-1, $a' \leq b$ and the columns h_i's in the second bracket are any b or less consecutive columns among the first (j-1-a) columns.

This condition ensures that there shall not be a code vector which can be expressed as sum (difference) of two solid bursts of length b or less each. Thus, the coefficients u_i form a solid burst of length a and the coefficients v_i form a solid burst of length b or less in a (j-1-a)-tuple.

The number of choices of these coefficients is given by (refer [4])

$$\sum_{i=1}^{b}\sum_{l=1}^{b}(j-l-i+1)(q-1)^{i+l-1}. \qquad (3)$$

Now according to condition (ii), the syndrome of any solid burst error of length $s(>b)$ or less must be nonzero and different from those syndromes resulting from solid burst error of length b or less.

In view of this, h_j can be added provided that

$$h_j \neq (u_{j-1}h_{j-1} + u_{j-2}h_{j-2} + \ldots + u_{j-s'+1}h_{j-s'+1} + u_{j-s'}h_{j-s'}),$$

where $s' \leq s$ and the coefficients u_i's are non-zero.

All the linear sums of b-1 or less of the above expression are already included in (3), therefore the coefficients u_i's can be chosen as follows:

$$\sum_{i=1}^{s-1}(q-1)^i - \sum_{i=1}^{b-1}(q-1)^i = \sum_{i=b}^{s-1}(q-1)^i. \text{ (refer [4])} \qquad (4)$$

Thus the total number of linear combination to which h_j can not be equal is

(3) + (4).

At worst all these combinations might yield distinct sum.

Therefore h_j can be added to H provided that

$$q^{n-k} > 1 + (3) + (4).$$

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Or, $$q^{n-k} > 1+\sum_{i=1}^{b}\sum_{l=1}^{b}(j-l-i+1)(q-1)^{i+l-1}+\sum_{i=b}^{s-1}(q-1)^{i}.$$

Replacing j by n gives the theorem.

4. AN ILLUSTRATION

Consider a (7, 2) binary code with the 5×7 matrix H which has been constructed by the synthesis procedure given in the proof of theorem 2 by taking $q = 2$, $b = 2$, $s = 4$, $n = 9$.

$$H=\begin{bmatrix}1&0&0&0&0&1&0\\0&1&0&0&0&0&1\\0&0&1&0&0&1&0\\0&0&0&1&0&0&1\\0&0&0&0&1&1&0\end{bmatrix}$$

The null space of this matrix can be used to correct all solid bursts of length 2 or less and simultaneously detect all solid bursts of length 4 or less. It may be verified from error pattern-syndromes table 1 that the syndromes of all solid bursts of length 2 or less are non zero and distinct and syndromes of all solid bursts of length 4 or less are non zero and different from solid bursts of length 2 or less.

Table 1: Error patterns and corresponding syndromes

Error patterns	**Syndromes**	**Error patterns**	**Syndromes**
Solid bursts of length 1		Solid bursts of length 3	
1000000	10000	1110000	11100
0100000	01000	0111000	01110
0010000	00100	0011100	00111
0001000	00010	0001110	10110
0000100	00001	0000111	11110
0000010	10101	Solid bursts of length 4	
0000001	01010	1111000	11110
Solid bursts of length 2		0111100	01111
1100000	11000	0011110	10010
0110000	01100	0001111	11100
0011000	00110		
0001100	00011		
0000110	10100		
0000011	11111		

5. DISCUSSION AND CONCLUSION

This paper presents the bounds on parity checks for codes capable of correcting and simultaneously detecting solid burst errors, also deals with the construction of

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such codes. The bounds will determine the capability of error-detecting and -correcting of the codes. These bounds will be useful to combat solid burst error where both error detection and correction is required.

The optimal codes are very useful from application point of view in communication as having minimum redundancy and improving the rate of transmission. Therefore optimal codes dealing with the errors discussed in this paper and no other errors can be good work. Bounds similar to the ones obtained in this paper w.r.t. the metric studied by Kitakami et al. [12] may also be derived.

REFERENCES

[1] Argyrides CA, Reviriego P, Pradhan DK, Maestro JA. Matrix-Based Codes for Adjacent Error Correction. *IEEE Transactions on Nuclear Science.* 2010; 57(4): 2106-2111.

[2] Arlat J, Carter WC. Implementation and Evaluation of a (b,k)-Adjacent Error-Correcting/Detecting Scheme for Supercomputer Systems. *IBM J. RES. DEVELOP.* 1984; 28(2): 159-169.

[3] Bossen DC. *b*-Adjacent Error Correction. *IBM Journal of Research and Development*. 1970; 14(4): 402-408.

[4] Das PK. Codes Detecting and Correcting Solid Burst Errors. *Bulletin of Electrical Engineering and Informatics.* 2012; 1(3): 225-232.

[5] Das PK. Codes Detecting and Locating Solid Burst Errors. *Romanian Journal of Mathematics and Computer Science*. 2012; 2 (2): 1-10.

[6] Das PK. Blockwise Solid Burst Error Correcting Codes. *International Journal on Information Theory*. 2012; 1(1): 11-17.

[7] Dass BK, Verma R. Repeated burst error detecting linear codes. *Ratio Mathematica — J. Appl. Math.* 2009; 19: 25-30.

[8] Etzion T. Optimal codes for correcting single errors and detecting adjacent errors. *IEEE Trans. Inform. Theory*. 1992; 38: 1357–1360.

[9] Hamming RW. Error-detecting and error-correcting codes. *Bell System Technical Journal*. 1950; 29: 147-160.

[10] Jensen DW. Block code to efficiently correct adjacent data and/or check bit errors. Patent no: US 6604222 B1. Date of Patent Aug 5*(www.google.com/patents/US6604222)*. 2003.

[11] Katti RS. Comments on A Systematic (16, 8) Code for Correcting Double Errors and Detecting Triple Adjacent Errors. *IEEE Trans. on Computers*. 1995; 44(12): 1472-1473.

[12] Kitakami M, Jiang S, Fujiwara E. *Metrics of error locating codes*. Proc. of 1997 Int. Symp. Infom. Theory. 1997; 264.

ISSN: 2252-4274 (Print)
ISSN: 2252-5459 (Online)

[13] Peterson WW, Weldon EJ(Jr). *Error-Correcting Codes*, 2nd edition, The MIT Press, Mass, 1972.
[14] Ray-Chaudhuri DK, Singhi NM, Sanyal S, Subramanian PS. Theory and Design of t-Unidirectional Error-Correcting and d-Unidirectional Error-Detecting Code. *IEEE Trans. on Computer.* 1994; 43(10): 1221-1226.
[15] Sacks GE. Multiple error correction by means of parity-checks. *IRE Trans. Inform. Theory IT*. 1958; 4: 45-147.
[16] Schillinger AG. A class of solid burst error correcting codes. *Polytechnic Institute of Brooklyn*. N.Y., Research Rept. PIBMRI. April 1964; 1223-64.
[17] Sharma BD, Dass BK. Adjacent error correcting binary perfect codes. *J. Cybernetics*. 1977; 7: 9-13.
[18] Shiva SGS, Sheng CL. Multiple solid burst-error-correcting binary codes. *IEEE Trans. Inform. Theory*, IT-15. 1969; 188-189.

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