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Application of the Fundamental Solution Method to Object Recognition in The Pictures

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ABSTRACT

Recognition of objects in pictures and movies requires the use of techniques, such as filtering, segmentation and classification. Image filtering is required to remove all artifacts that hinder the unequivocal identification and sharpen interesting objects. Segmentation refers to finding areas of images respected to individual objects. For the selected areas corresponding to objects in the selected picture, the classification of objects finally gives information about the type of object which orientation is made. This paper presents a method for the classification of objects from drawings as a bitmap using the method of fundamental solutions (MFS). The MFS was tested on the selected bitmap depicting simple geometric shapes. The correlations between errors occurring on the boundary for particular shapes are used for the selection of geometric shape Figures. Due to this correlation, it is possible to recognize the shape of the image appearing on the drawing by an analysis consisting of the comparison of recognized points describing the shape of contour to a database containing solutions of boundary value problems for the selected shape. In one way, the comparison of the pattern can determine which shape from database it is most similar to in terms of contour. This article appear that this approach is very simple and clearly. In result, this method can be used to recognition of the objects in the systems of real-time processing.

Keywords*:* Image Analysis, Shape Recognition, Partial Differential Equation, Fundamental Solution Method.

1. INTRODUCTION

In the literature, it is possible to find more examples of applications and methods for the recognition of shapes. The one of the method is the automatic control of the image-smoothing strength in the MRF function to increase the performance of edge-preserving image smoothing algorithm [1]. In this method, the parameter control of the function depends on the edge magnitude resulting from discontinuities of image intensity and good representations of strong edges that might be the objects in the pictures. Different approaches to appearance-based object recognition and pose estimation is based on the images which are considered as high-dimensional vectors defining features which are transformed in unrestricted way [2]. The authors mainly concentrate on the comparison of preprocessing operations in the context of object recognition. The experimental evaluation provides recognition rates and poses estimation accuracy and finally, based on a standard image database. The one of the problem is concentrated on the edge recognition. Two methods for the edge detection is presented in [3]. These methods are based on the self-organizing map (SOM) and a grayscale edge detector, which are using to find details on the images which consist of meta-metric colors.

Two types of Partial Differential Equation (PDE), parabolic equations and elliptic equations, are the most widely used in recognition of the shape based on parabolic equations [4]. The image completion operation can be realized based on the Poisson equation [5]. One of the popular problems in image processing is removing significant objects from natural images or photographs. The process of reconstruction of images is typical realized based on the method repairing individual pixels in its surroundings but also the images are reconstructed from the gradient maps by solving a Poisson equation [6].

A more difficult problem to shape recognition occurs in the case of the analysis of 3D spaces. One of the most important examples of image analysis and computer vision in recent years are the methods used to face recognition [7]. Fast algorithms for the segmentation of 3D images are needed in applications such as medicine. On example the framework to describe 3D shapes is based on modeling the probability density of a shape functions [8]. On other hand the Poisson histogram is created as the shape descriptor in which the Poisson equation is used to define a 3D shape signature. The result shows that the Poisson histogram is good for the making of shape descriptors [9,10].

The main goal of this article is focused on the problem of shape recognition for which the contour is described by the elliptic differential equation. The implementation of the Poisson equation is used for the formulation of the boundary value problem, in which the boundary conditions are defined along the considered contour. One of popular methods to solve this problem is the finite difference method [11]. The Poisson equation is a class of partial differential equations which describe a steady-state temperature distribution in a bounded object. In [12] the Fourier equation is used for the modeling of image textures by constructing specific heat source functions and boundary conditions. The heat source function is considered as an image transform function, such that a set of texture features at different frequencies and orientations is extracted from the transformed image, in conjunction with the use of a Gabor wavelet filer bank.

A better performance of image texture retrieval by these features is achieved than while using the features extracted directly from the original image texture. The inhomogeneous Poisson equation with internal Dirichlet boundary conditions has recently appeared in several applications ranging from image segmentation, image colorization, digital photo matting and image filtering the object detection approach based on local shape information, used the boundary fragment extraction method using Poisson equation properties is presented in [13]. In paper [14] there is presented the algorithm for a segmentation-based detection. The proposed method assumes that the shape of each segment is described both by its significant boundary sections and by regional and dense orientation information derived from the segment shape using the Poisson equation.

Apart from other methods to solve BVP problems, the Method of Fundamental Solution (MFS) is a very interesting method. This method is a specific type of Trefftz method, for which the solution is given as linear combination of functions that are the fundamental solution. For the solution of BVP problems, apart from

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collocation points on the boundary, there is required the congregation of source points outside consideration area.

2. MATERIAL AND METHODS

The MFS is a boundary type method suitable for the solution of certain elliptic boundary value problems. The basic ideas of the MFS were introduced by Kupradze and Alexidze. In work [15] there was investigated the influence of the position of source points on the boundary error. Paper [16] consists of the presentation of the development of the MFS and several applications of MFS-type methods. There are also outlined techniques by which such methods are extended to certain classes of non-trivial problems and adapted for the solution of inhomogeneous tasks.

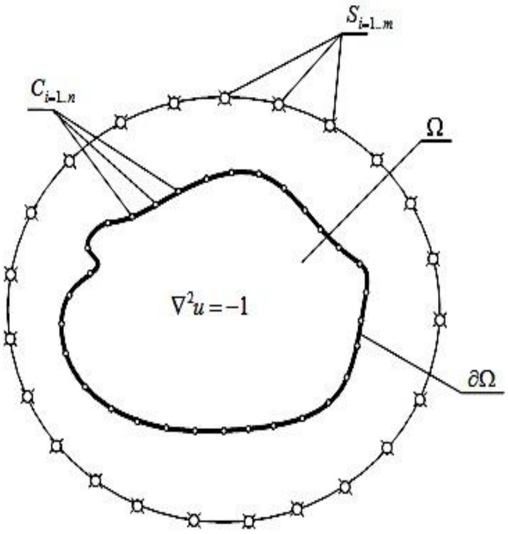

FIGURE 1. Placement of source and collocation points in consideration area

In the MFS, the problem is defined by the congregation of collocation points, which are placed around the edge of a selected object. The number of collocation points may have any value. For calculation, this method required the congregation of a source, which can be placed along the area of consideration, outside this area. The second popular type of the placement of source points is the method, for which points are placed along a circle inside which we can find the area of consideration. The second method is better for irregular shapes, because the generation of these points is simpler for concave shapes.

The problem of recognition of image objects can be solved as the Cauchy problem, in which the governing equation has the form of,

$$\nabla^2 u = g(x, y) \quad \text{in} \quad \Omega \tag{1}$$

with boundary conditions,

$$u = 0 \quad \text{on} \quad \partial\Omega \tag{2}$$

The general solution can be obtained as a sum of particular and homogenous solution,

$$u = u_p + u_h \tag{3}$$

The right side of Equation (1) is assumed as the function $g(x,y) = -1$, and the particular solution must be fulfilling governing equation. As a result, the particular solution is obtained from the equation:

$$\nabla^2 u_p = -1 \tag{4}$$

As a result of integration of the Equation (3) the particular solution is defined as:

$$u_p = \frac{1}{4}\left(x^2 + y^2\right) \tag{5}$$

The homogenous solution is assumed in the form:

$$u_h = \sum_{i=1}^{m} c_i Ln(r_{i,j}) \quad , \tag{6}$$

where $r_{i,j}$ is defined as distance between source and collocation points:

$$r_{i,j} = \sqrt{\left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2} \tag{7}$$

The coefficients c_i can be determined using the boundary collocation method. Equation (6) fulfills the governing equation (1) and must fulfill boundary conditions for all collocation points. The existence condition of solution for set of equations creating based on Equation (6) for all collocation points is that the number of collocation points must be equal or greater than number of source points. If the number of source and collocation points is equal, finding unknown coefficients c_i comes down to solving the system of equations. In case where the number of collocation points is greater than source points the solution comes down to finding an approximate solution in the least square sense.

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2.1 NORMALIZATION METHOD

The case in which the shapes are of different sizes the normalization process should be carried out. The method of normalization is realized in two steps. In the first one, for shape points, the circle in the least square approximation is determined. The circle equation is defined as,

$$(x_i - x_s) + (y_i - y_s) = r^2 \tag{8}$$

Using the substitution $\lambda = -r^2 + x_S^2 + y_S^2$ the Equation (8) has the form,

$$-2x_i x_s - 2y_i y_s + \lambda = -x_i^2 - y_i^2 \tag{9}$$

The set of equations is created for all shape points $P_i = \{(x_1, y_1),..., (x_i, y_i)\}$ for which the unknown values of x_S, y_S, λ are calculated. Substitution λ in (9) makes this equation as linear. The unknown parameters x_S, y_S, r defined the center and radius of circle. They are obtained using least the square method.

In the second step, the points P_i are scaled to points Q_i for which the radius is equal 1 according to equation:

$$Q_i = \begin{cases} \hat{x}_i = x_s + \frac{1}{r}(x_i - x_s) \\ \hat{y}_i = y_s + \frac{1}{r}(y_i - y_s) \end{cases} \tag{10}$$

The scaling method is used to normalization of points and in the results. The contour described analyzing object can be compared with objects including in database.

2.2 IMAGE RECOGNITION PROBLEM

The presented method was used to shape recognition for 8 simple monochrome pictures presented on Figure 2. To test has been selected shapes in the form of a bitmap in which objects are represented by a black pixel color on a white background. Such statement of object and background colors allows easy determination of objects edges. It is not a restriction for presented method, because there is known edge detection methods allow to definition the outline of images shot in full color palette.

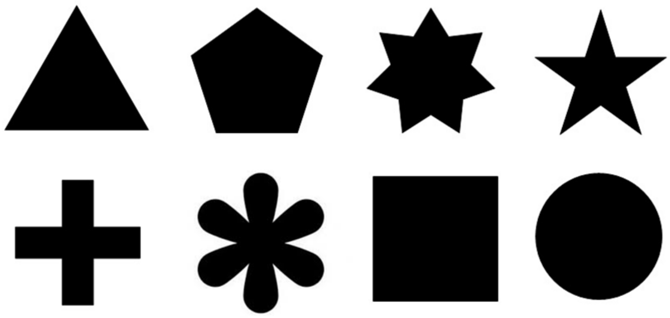

FIGURE 2. Simple Figures for recognition.

For selection of edges for each picture there was used the morphological filter called Sobel Edge. Adaptive edge detection approach based on the image context analysis is presented in [17]. The experimental results indicate that both visual evaluations and objective performance evaluations of the detected image in this approach are superior to the edge detection. Details and the scheme of procedure is presented by Tsai et al. [18]. The proposed approach by Tsai, which is far more accurate than the detection scheme, can precisely locate object contours in the image, especially for complex scenes. This feature, which is lacking in the edge detection scheme, is of extreme importance to some applications, such as data hiding, watermarking, morph, and pattern recognition. In addition, the approach can be integrated into a prediction-based lossless image compression scheme to provide both the lossless compression codes and edge maps of objects, which facilitate the image transmission and object recognition for medical diagnoses and other applications. Hui et. al. [17] puts forward an effective, specific algorithm for edge detection based on multi-structure elements of gray mathematics morphology, in the light of a difference between the noise and edge shape of images.

The authors establish multi-structure elements to detect edge by utilizing the grey form transformation principle. Compared to some classical edge detection operators, such as Sobel Edge Detection Operator, LOG Edge Detection Operator, and Canny Edge Detection Operator, the experiment indicates that this new algorithm possesses a very good edge detection ability, which can detect edges more effectively, but its noise-resisting ability is relatively low. The morphologic edge detection methods using multi-scale approach for detecting edges of various fineness under noisy condition is shown in [19], where proposed edge detector has the desirable properties that a good edge detector should have. The comparative study reveals its superiority over other morphologic edge detectors.

One of simple and popular approaches to the edge detection method is connected with the definition of an objective function which is based on pixels inside the 3x3 mask matrix consisting of neighboring the pixels for the picture [20]. The values of the objective function corresponding to four directions determine the edge intensity and edge direction of each pixel in the mask. After all pixels in the image have been processed, the edge map and direction map are generated. The proposed method can detect the edge successfully, while double edges, thick edges, and speckles can be avoided.

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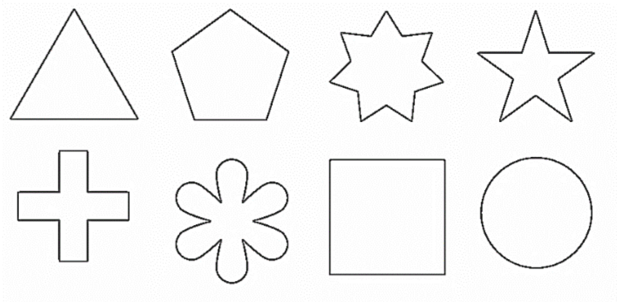


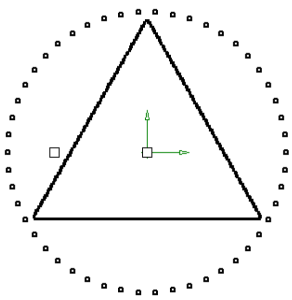
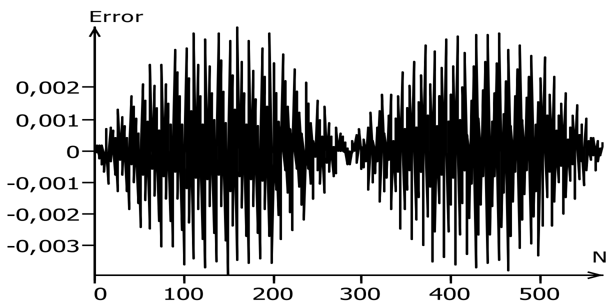
FIGURE 3. Contours selected by Sobel edge method

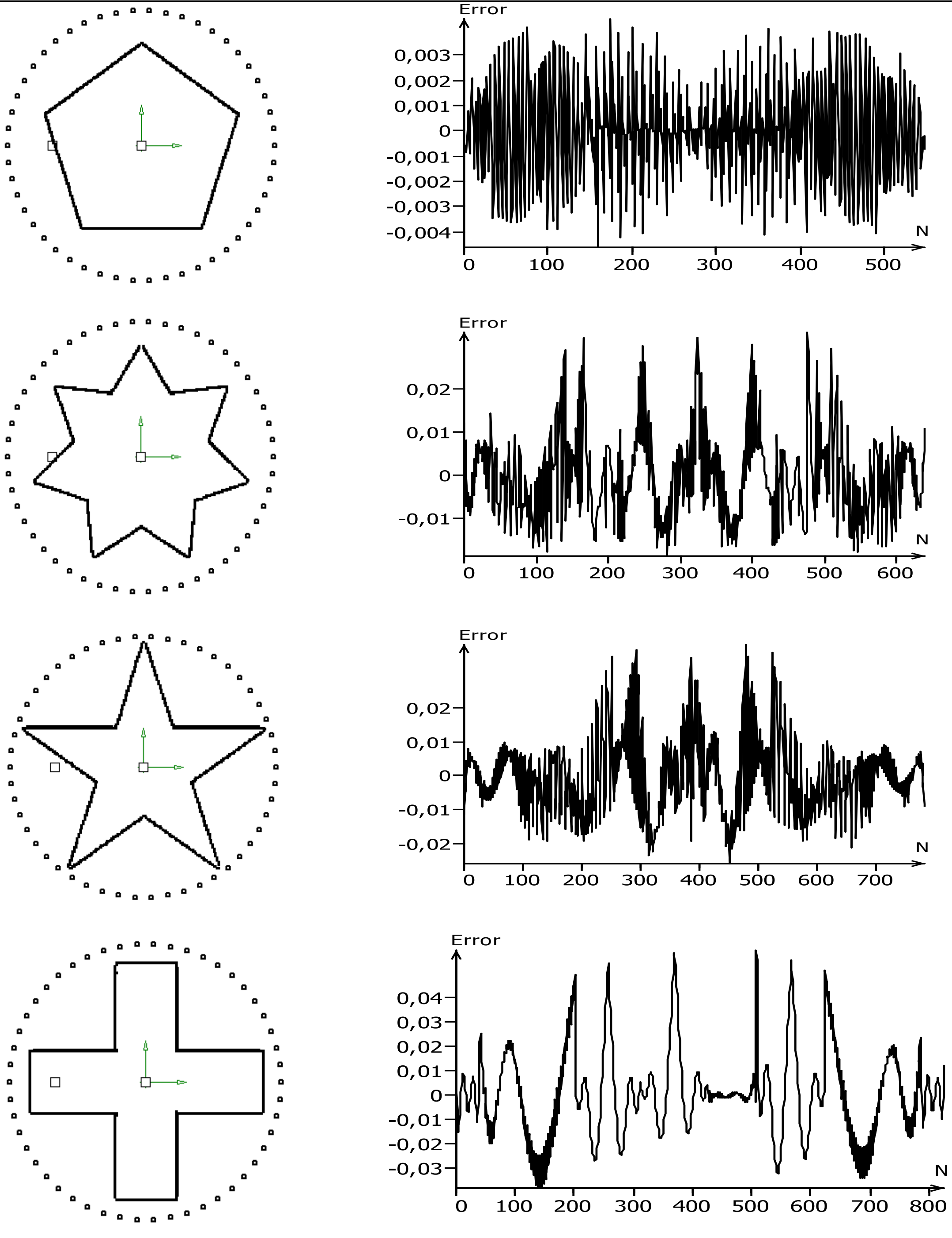
In the presented approach to selection of the edges the Sobel Edge, the morphological filtering method has been used. After filtering the selected points, with coordinates equivalent to pixel position on the image, they are used for normalization. The normalization process is needed for the possibility of comparison of each different Figures. The examples of picture (Figure 3) are characterized as closed loop edges. The shape points can be written to the hypothetical cycle defined by the position of the center point and its radius. The proposed method of calculation of parameters gives very good results in comparison to a solution based on the calculation of the average value of coordinates applied to marker detection [21].

3. RESULTS

Table 1 presents solutions for each of 8 pictures. The first column presents the location source and collocation points. The second column of the Table consists of a graph of the error on the boundary around the edge. The mean value of the error depends on a shape. For objects, for which the shape is convex, the error is lesser.

TABLE 1.
Error on boundary of particular examples

Kind of shape	Error on boundary
	



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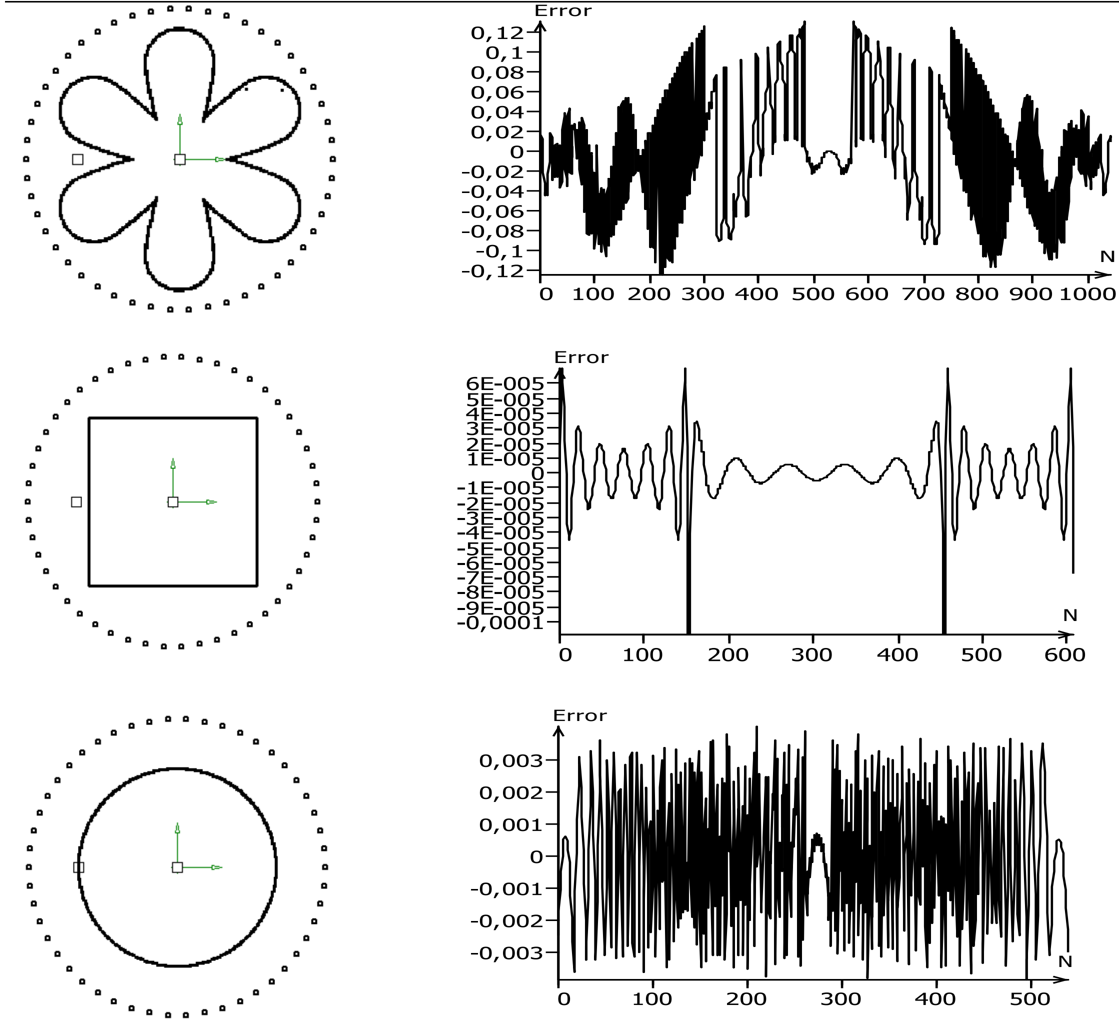

For each example the obtained solution was used to solve the error on the boundary in comparison to each of the other solutions. Table 2 presents errors for all eight shapes. The error on the boundary defining the similarity of the shape because the MFS method approximate solution of governing equation on the boundary. As a result, for the same shapes represented by a set of the points distributed on similar regions give the minimal solution error on the boundary in BVP. In theory, for the same shapes, the error on the boundary should equal 0, but in practice this value depends on some parameters, such as the shape of a region, the number of a collocation and source points, and the placement of source points [22].

TABLE 2.
Correlation between error values for particular shapes

No	Kind of Shapes							
	1	2	3	4	5	6	7	8
1	**0,002**	0,117	0,100	1,048	0,190	0,212	0,081	0,099
2	0,094	**0,003**	0,070	0,071	0,135	0,118	0,053	0,028

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3	0,418	0,090	**0,009**	0,415	0,257	0,231	0,109	0,063
4	0,556	0,077	0,268	**0,009**	0,416	0,245	0,113	0,140
5	0,489	0,132	0,130	0,605	**0,015**	0,459	0,608	0,139
6	0,044	0,075	0,089	0,218	0,232	**0,051**	0,064	0,093
7	0,102	0,054	0,080	0,134	0,172	0,118	**0,001**	0,049
8	0,121	0,031	0,060	0,105	0,120	0,113	0,050	**0,003**

The correlations between particular shapes presented in Table 2 showed that, for the same shapes, the error on the boundary is clearly lesser than for other shapes. The obtained results suggest the proposed method for any to known shapes from the properly prepared database.

4. CONCLUSIONS

The method presented in this article is very useful for recognition of any object from the pictures. The base of the proposed method is the definition of a shape recognition problem as the BVP problem. The proposed method for solution of BVP is not complicated and relatively fast, so this method can be used for the recognition of shapes in the movies in real time. The proposed procedure of recognition of objects needs a database of known shapes. For identification of shapes, the contour of objects has been described by Poisson equation. The method of image recognition has been defined as Boundary Value Problem and solved by a simple method called the Method of Fundamental Solution. This method has an advantage that is computationally undemanding. The comparison of contours is realized by the calculation of linear combination of functions representing the solution of the Laplace equation. Ride side of governing equation was added as exact solution for the Laplacian equation equal 1. The adopted method of solving the boundary problem also has a relatively simple way to perform the image analysis requiring transformation in the form of scaling and rotation. This transformation, realized only for source points, consequently applicable to pattern shape.

Using a suitable database of known shapes of the analyzed image can be subjected to a comparison to determine the similarity. As the degree of similarity value the error of collocation boundary conditions is adopted. The presented method to compare shapes consisted of the calculation error at the edge of the contour shows that in case of the matching shapes pattern for the analyzed image, the error at the edge is rapidly decreasing. Accordingly, such an approach seems to be interesting from the point of view of the application of the image analysis.

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