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Optimal Control of Jebba Hydropower Operating Head by a Dynamic Programming

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ABSTRACT

Nigeria with a generating potential of roughly 12,522 MW only supplies less than 20% of the national demand. This necessitates an optimal use of the Jebba Hydroelectric Power Plant whose optimal generation depends on the operating head. This paper presents the solution to an optimal control problem involving the operating head of the plant. An optimal control problem consisting of a model of the system dynamics, performance index and system constraints was solved using a dynamic programming approach. The control procedure was built on the integration of the nonlinear dynamical model by an Adams-Moulton technique with Adams-Bashfort as predator and Runge-Kutta as a starter. The numerical solution, coupled with dynamic programming was employed in developing an optimal control procedure for the regulation of the operating head. Result presented shows the potential of the control procedure in determining the amount of inflow required to restore the operating head to a nominal level whenever there is a disturbance.

Keywords: Dynamic programming, Hydropower, Inflow, Operating head, Optimal Control.

1. INTRODUCTION

Electricity generation in Nigeria has been lower than required and the supply is epileptic. Although the nation is powered from roughly 29 generating stations but the power available on the grid is every non-sufficient. As at 30th of January 2019, the peak generation was 4,328.40 MW as against the installed capacity of 12,910 MW. Both the generation and installed capacity are lower to the estimated national peak demand forecast of 23,020 MW, hence the generation capacity of each station should be optimized continually to supply the population estimated at 185,989,640.

Jebba Hydroelectric powers station JHEPS is one of the reliable generating facilities of the nation, it contributes appreciably percentage of the installed generating capacity. It is located on the River Niger at latitude 09^0 08^' 08^" N and longitude 04^0 47^' 16^" E with an installed capacity of 578.4 MW form six units of 96.4 MW turbo-alternator [1]. JHEPS play an important role in both supplying and stabilizing the national grid. It has high reliability as compared to the thermal stations and the cost of generating energy is cheaper as well.

In Sambo et al. [2], a major factor affecting power generation in the country is a poor utilization of the existing resources. Therefore, it is imperative to explore and design efficient means of obtaining a reliable and optimal power from the JHEPS. The JHEPS Reservoir depends on discharge and spill from the Kainji Hydroelectric Power Station (KHEPS). This necessitates a formal resources management and control procedure such that the units at JHEPS can operate throughout the year. Unfortunately, operators are faced with challenges involving the operational safety of the stations and the power demand requirements from the grid [3], [4]

KHEPS and JHEPS operate in cascade but there is no control system to regulate their operation. The reservoir head is being managed by observation of inflows and experience[5]. From the daily operational report form the TCN NCC, it is obvious that the potential of JHEPS has not been maximised. There are occasions where some units at JHEPS are shut down if the release from KHEPS is low. There has been research on the forecast and management of inflow such as to ensure optimal power generation [6]. Meanwhile, the regulation of the reservoir head does not depend on the inflow alone but also the unit's availability and environmental factors that are weather-related. The optimal determination of the amount of release required from KHEPS for the regulation of JHEPS remains a potential problem unsolved.

From literature, the have been numerous work on optimization of water resources for optimum system performance and economic benefit [7]. In most cases, the solution to the optimization problem is difficult because of the large set of variables involved and nonlinearity of system dynamics. As a result, there exist several mathematical programming techniques but most only solves a particular class of problem. A method of handling a general form of reservoir optimization problem does not exist [7][8]. A given type of optimization problem may, therefore, require an optimization technique.

In this work, the optimal regulation of JHEPS reservoir head is posed as an optimal control problem. The system dynamical model was presented in [9] to satisfy a nonlinear differential equation. The performance index is the minimization of deviation of the head from the nominal value while the constraints include the system dynamical model. Since the model, the performance index and the associated constraints are nonlinear, hence the solutions to such optimal control problem become more challenging.

Optimal Control is the determination of the control signal and the state trajectories for a dynamical system, within an interval of time, in order to minimise a given performance index [10]. Unfortunately, many problems that are rooted in nonlinear optimal control theory do not have computable solutions or they have solutions that may be obtained only with a great deal of computing effort [11].

Standard theories of optimal control are presented in [12-14]. Solutions to optimal control problems are broadly categorized into two: the direct and indirect methods. In the direct methods, the optimal solution is obtained by direct minimization of the performance index subject to constraints. The indirect method applies calculus of variation to set up necessary conditions that must be satisfied by the optimal control. Calculus of the, together with Pontryagin's minimum principles are used to setup optimality conditions. These conditions produce optimal control canonical equations such that their solution ensures that an optimum point has been reached. While using this approach, it is usually necessary to calculate the Hamiltonian, co-state

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equations, the optimality and transversely conditions (Rodrigues et al., 2014). The difficulty in solving optimal control problem by indirect methods is that it is necessary to calculate the Hamiltonian, adjoint equations, the optimality and transversely conditions. The approach is also not flexible, a new derivation is required whenever a new problem is formulated [10].

Hydropower system has large numbers of nonlinearities and stochastic variables, hence they are usually optimized by dynamic programming. The solution to an optimal control problem using dynamic programming is based on the Bellman Principle of Optimality. The principle of optimality states that an optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision [13][15][16].

Dynamic programming is a multi-stage procedure that determines the best combination of decision variables that optimize a certain performance index. It has a potential of optimizing convex and non-convex, linear and nonlinear, continuous and discrete and a constrained and unconstrained system. These are what makes it superior to other technique [8].

In Sundström et al. [17], optimal control involving hybrid electric vehicle energy management was solved using dynamic programming. Similarly, an algorithm motivated by dynamic programming solved the determination of an optimum continuous input and optimal switch sequence for a two-stage optimization problem involving a switch system [18]. It was also used in [19][20] to solve optimal control problem relating to energy management of plug-in hybrid electric vehicles and management of a cascaded hydropower system. Therefore, this work employed dynamic programming to solve an optimal control problem involving the regulation of the operating head of JHEPS.

2. PROBLEM FORMULATION

Solution to an optimal control problem involves the determination of the control signal required to move a system state from an initial point to a desired time in finite time another Optimal control is purely an optimal control problem whereby a control signal is desired that will force the reservoir head at JHEPS to move from an initial point to the desired point in finite time and subject to constraints imposed by the system dynamics.

2.1 JHEPS DYNAMICAL MODEL

Figure 1 the schematic diagram of the JHEPS where h is the reservoir operating head (m), Q is the inflow into the reservoir (m^3/s), Q_L is the losses on the reservoir that is majorly due to evaporation (m^3/s), Q_s represent the flow through spillway (m^3/s), q is the inflow along the penstock (m^3/s), A_1 is the effective surface area of the reservoir (m^2) while $A_2$is the area of the inlet into the scroll casing (m^2). q_k and Q_{sk} are the discharge and spill from KHEPS respectively, while Q_{CJ} is the inflow within the catchment area between Kainji and Jebba hydropower stations.

The total power generated from the station can be expressed as Equation 1 [9], where P is the power generated by the station, n is the number of units in operation (n can take an integer value from 1 to 6), η represents the conversion efficiency of the turbo-alternator and g is the acceleration due to gravity.

$$\boldsymbol{P} = \sqrt{2}\ n\eta\rho A_2 g^{3/2} h^{3/2}\ (\mathrm{W}) \quad (1)$$

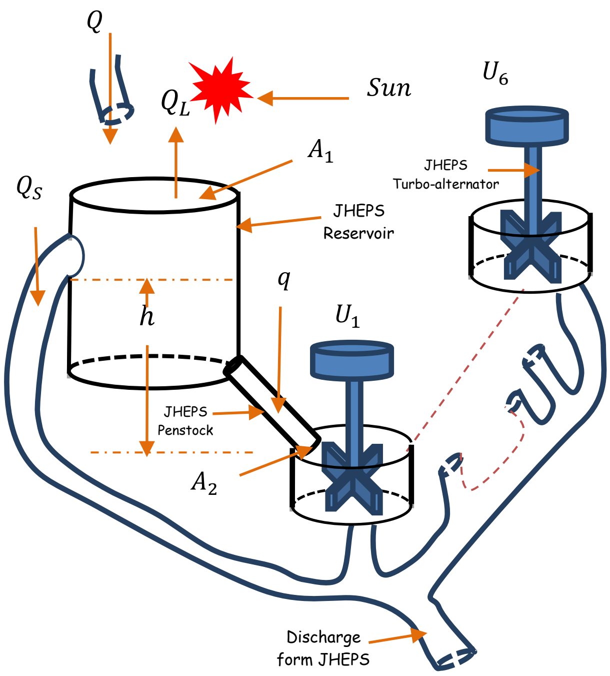

FIGURE 1. Schematic Diagram of JHEPS

Equation 1 shows that the power dynamics is a function of the operating head dynamics, of which the dynamical model equation can be represented by Equation 2.

$$\frac{dh(t)}{dt} = -n \propto h^{1/2}(t) + \mu u(t) \quad (2)$$

$$\alpha = A_1^{-1} A_2 \sqrt{2g}\ ;$$

$$\mu = A_1^{-1}$$

$$u(t) = q_k(t) + Q_{sk}(t) + Q_{CJ}(t) - Q_L(t) - Q_s(t) \quad (3)$$

Equation 2 is presented in a standard form as

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$$\dot{h}(t) = f(h(t), u(t); t) \quad (4)$$

$$h(t_0) = h_0$$

$$t \in [t_0, t_f]$$

2.2 THE SOLUTION OF THE NONLINEAR MODEL

The essence of the model equation is for it to be used in control system design, hence the solution is desired. It is evident that the dynamical model satisfies a nonlinear differential equation from which the closed-form solution is not readily available, hence a numerical solution has to be employed.

The numerical solutions employed is the Adams Moulton numerical techniques with Adams Bashfort as predictor and Runge Kutta starter. The procedure is as follows:

Given that $h_{(n-4)} = h_0$ and $u(t)$ is also specified.

By Adams – Moulton numerical technique, h_n can be expressed as equation (). The predictor $\widetilde{h_n}$ is computed by Adams – Bashforth technique of equation (). Since Adams-Moulton technique is not self-starting, the intimidate head ($h_{(n-3)}$, $h_{(n-2)}$, $h_{(n-1)}$) were computed using the Runge - Kutta technique of equation (5):

$$h_n = h_{(n-1)} + \frac{\Delta t}{24}(9f(\tilde{h}_n) + 19f(h_{(n-1)}) - 5f(h_{(n-2)}) + 9f(h_{(n-3)}) \quad (5)$$

$$\widetilde{h_n} = h_{(n-1)} + \frac{\Delta t}{24}(55f(h_{(n-1)}) - 59f(h_{(n-2)}) + 37f(h_{(n-3)}) - 9f(h_{(n-4)}) \quad (6)$$

$$h_{(n-3)} = h_{(n-4)} + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \quad (7)$$

where,

$$k_1 = \Delta t * f(t_n, h_{(n-4)}) \quad (8)$$

$$k_2 = \Delta t * [f(t_n, h_{(n-4)}) + 0.5k_1] \quad (9)$$

$$k_3 = \Delta t * [f(t_n, h_{(n-4)}) + 0.5k_2] \quad (10)$$

$$k_4 = \Delta t * [f(t_n, h_{(n-4)}) + k_3] \quad (11)$$

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3. OPTIMAL CONTROL PROBLEM

The optimal control problem solved in this work is the determination of the control vector $u(t) \epsilon U(t)$, which compels the dynamical system $\dot{\boldsymbol{h}}(t) = f(h(t), u(t), t)$ to follow the optimal trajectories $h^*(t)$ that minimize specified performance indices(J).

The selected performance index was selected to be quadratic and defined as Equation (12);

$$J = min \int_{t_0}^{t_f} [h(t) - h_T]^T \boldsymbol{K_h} [h(t) - h_T] dt \tag{12}$$

Subject to the system constraint;

$$\dot{h}(t) = \boldsymbol{f}(h(t), u(t), t) \qquad ; \ t_0 \leq t \leq t_f$$

$$h(t_0) = h_0$$

$$h_{t_f} = h(T)$$

$h(T)$ is the desired final value for the state vector while $\boldsymbol{K_h}$ is a positive definite weighting matrix or scalar constant.

3.1 DISCRETIZATION OF CONTROL AND STATES

In order to use the dynamic programming approach, the continuous system is approximated to a discrete system by quantizing the admissible control, state and time to a finite level. This was achieved by subdividing the time interval into $N > 1$ control state:

$$t_0 < t_1 < t_2 < \cdots < t_N = t_f.$$

In each of the sub-interval, (t_{i-1}, t_i) approximate $u(t) = u_i(t)$, such that:

$$U(t) = [u_1, u_2, u_3, \dots \dots u_N].$$

The problem can then be solved using dynamic programming as the optimization method.

3.2 A DYNAMIC PROGRAMMING SOLUTION

As earlier mentioned, dynamic programming algorithms are effective in solving problems related to control of multi-reservoir system operation. To use the dynamic programming approach, the continuous control problem system is approximated by quantizing the admissible control, state and time to a finite level with optimal control required at each time interval.

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The control algorithm is based on Figure 2, where the time interval $t_0 \rightarrow t_f$ is partitioned into $[0\ \ T] = \ \bigcup_{k=0}^{N}\{t_k, t_{k+1}, \dots\}$.

Let k represents the current discrete state of the system, where $k = [0,1,2, \dots, N]$.

$\boldsymbol{u}_i^k(t)$ is the allowable control selected at k, $\boldsymbol{u}(t) \ = [\, u_1(t),\ u_2(t)] = [0,\ 1]$.

The state $k+1$ is the state adjacent to k which is reachable by application of $\boldsymbol{u}_i^k(t)$ at k.

The operating head $h(t_0) = \ h_o$ while the last operating head is as expressed as;

$$\boldsymbol{h}(N) = \ \boldsymbol{h}\left(\boldsymbol{u}_i^{k=0}(t),\ \ \dots,\ \boldsymbol{u}_i^{k-1}(t), \boldsymbol{u}_i^{k}(t),\ \boldsymbol{u}_i^{k+1}(t), \dots, \boldsymbol{u}_i^{N-1}(t)\right)$$

The performance index measure for moving from $k \rightarrow k+1$ by application of $\boldsymbol{u}_i^k(t)$ is represented by $\boldsymbol{J}\left(\boldsymbol{u}_i^k(t)\right)$.

$\boldsymbol{h}\left(\boldsymbol{u}_i^k(t)\right)$ represents the operating head at state k by application of $\boldsymbol{u}_i^{k=0}(t), \dots, \boldsymbol{u}_i^{k-1}(t)$.

$\boldsymbol{J}\left(\boldsymbol{u}_i^k(t),\ \boldsymbol{u}_i^{k+1}(t)\right)$ is the performance measure for moving from $k+1 \rightarrow k+2$ by application of $\boldsymbol{u}_i^k(t)$ at k and $u_i^{k+1}(t)$ at $k+1$.

$\boldsymbol{h}\left(\boldsymbol{u}_i^k(t),\ \boldsymbol{u}_i^{k+1}(t)\right)$ represents the head at state $k+1$ by application of $u_i^{k=0}(t),\ \ \dots, u_i^k(t)$.

The summation of min performance measure for moving from $k+1 \rightarrow N$ by application of $[u_i^{k+1}(t), \dots, u_i^{N-1}(t)]$ is $\boldsymbol{J}^{\boldsymbol{\sigma}}\left(\boldsymbol{u}_i^{k+1}(t), \dots,\ \boldsymbol{u}_i^{N-1}(t)\right)$.

$\boldsymbol{J}^{\boldsymbol{\sigma}}\left(\boldsymbol{u}_i^k(t), \dots, \boldsymbol{u}_i^{N-1}(t)\right)$: Summation of min performance measure for moving from $k \rightarrow N$ by application of $[\boldsymbol{u}_i^k(t), \dots, \boldsymbol{u}_i^{N-1}(t)]$.

From the principles of optimality;

$$\boldsymbol{J}^{\boldsymbol{\sigma}}\left(\boldsymbol{u}_i^k(t), \dots,\ \boldsymbol{u}_i^{N-1}(t)\right) = min\left\{\boldsymbol{J}\left(\boldsymbol{u}_i^k(t),\ \boldsymbol{u}_i^{k+1}(t)\right) + \boldsymbol{J}^{\boldsymbol{\sigma}}\left(\boldsymbol{u}_i^{k+1}(t), \dots,\ \boldsymbol{u}_i^{N-1}(t)\right)\right\}$$

$\boldsymbol{u}^*(t)$ is the set of controls $[\boldsymbol{u}_{\boldsymbol{i}}^{\boldsymbol{k}}(\boldsymbol{t}), \dots, \boldsymbol{u}_i^{N-1}(t)]$ resulting at $\boldsymbol{J}^{\boldsymbol{\sigma}}\left(\boldsymbol{u}_i^k(t), \dots,\ \boldsymbol{u}_i^{N-1}(t)\right)$

h^* is the resulting head from $\boldsymbol{u}^*(t)$.

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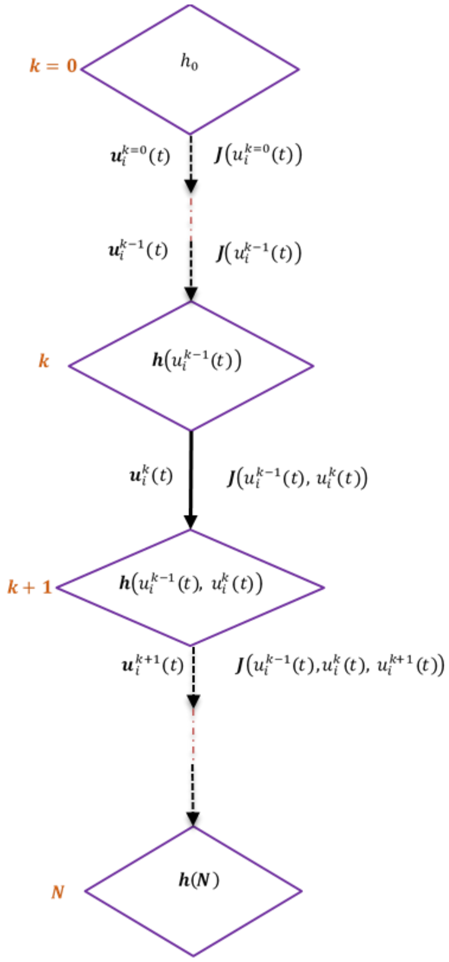

FIGURE 2. Optimal Control: Dynamic Programming Approach

4. RESULTS

The implementation and result of the procedure presented in section 3 are presented in this section. The dynamic programming algorithm was implemented in an EXCEL VBA® programming environment. Hence the algorithm can run on most commonly available machines, provided it has Microsoft Office installed.

Figure 3 illustrates the graphical implementation of the control procedure for the optimal control of JHEPS. Recall that $\boldsymbol{u}(t) = [u_1(t), u_2(t)] = [0, 1]$, where $u_1(t) = 2000\, m^3/s \equiv 0$ and $u_2(t) = 4000\, m^3/s \equiv 1$. The time interval is 24 hrs and this time is partitioned into four (6 hrs interval) discrete states represented by k. The operating condition has $h_0 = 25.8\, m$ and $h(T) = 26.1\, m$. The number of operating machines is 5 units.

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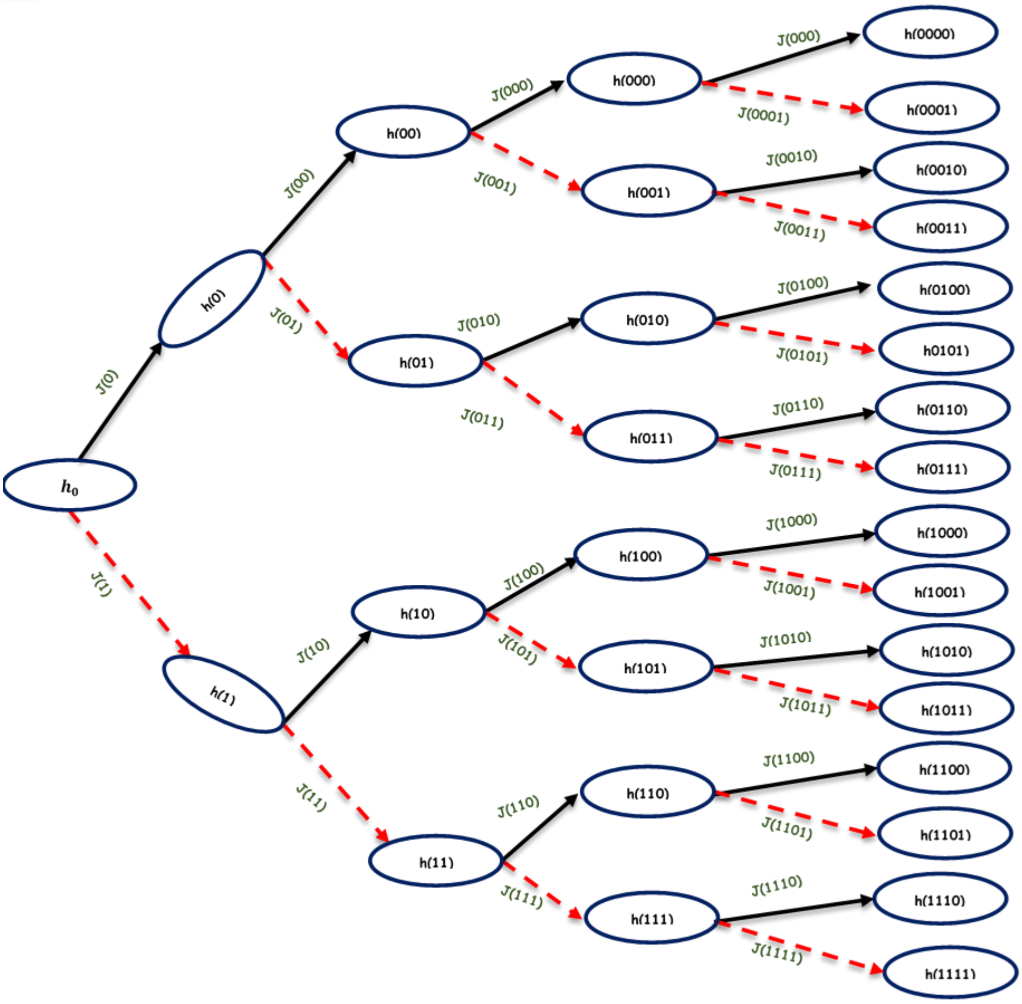

FIGURE 3. Layout Network of Optimal Control Design Based Dynamic Programming

The model was numerically integrated form $t = 0$ to state $k = 1$ with the two set of controls $u(t) = [0,1]$. In each case, the performance index was computed. This procedure was repeated until $k = N$. The result obtained is presented in Figure 4 showing the operating head at each state and the performance index relating to a control decision.

In other to compute the optimal path and hence optimal control, the feasible paths are presented in Table 1. Observation of Table 1 shows the optimal path to be in serial number 12. Similarly, Figure 5 shows the plot of the performance index against the associated paths. The optimum point, the performance index must be minimum. This is evident at the point marked **X** in Figure 5.

Hence, the optimal control required for moving the operating head of JHEPS from $h_0 = 25.8\, m$ to $h(T) = 26.1\, m$ in $24\, hr$ is by the release of inflow from KHEPS as follows: $4000\, m^3/s$ for the first six hours, $4000\, m^3/s$ in the second six hours, $2000\, m^3/s$ in the third and $2000\, m^3/s$ in the last six hours. Under this condition, the head trajectory is as presented in Figure 6.

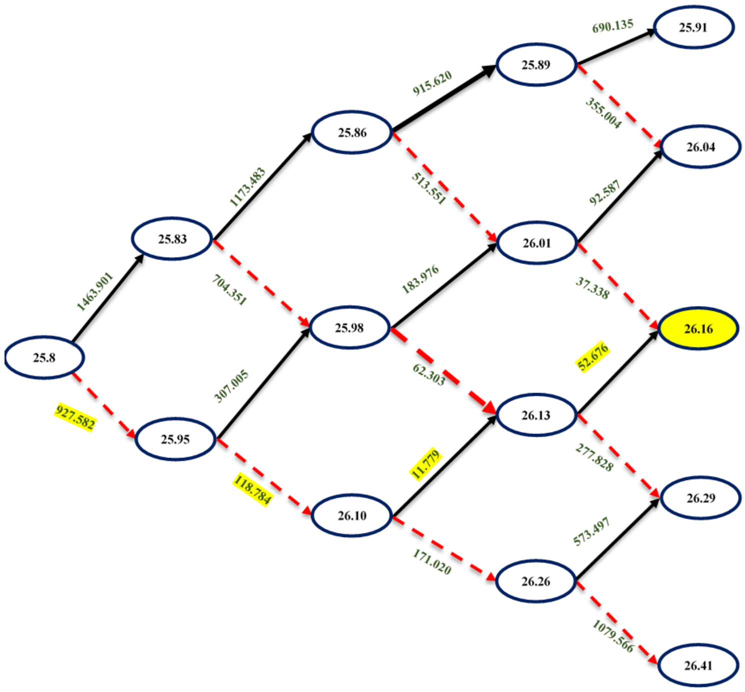

FIGURE 4. Dynamic Programming Solution to an Optimal control of JHEPS

TABLE 1.
Feasible Paths and Associated Performance Index

S/No	$u_i^k(t)$				$\sum_{k=1}^{4} u_i^k(t)$	J^k				$\sum_{i=1}^{4} J_i$
	k					k				
	1	2	3	4		1	2	3	4	
0	0	0	0	0	0	1463.9	1173.48	915.62	690.14	4243.139
1	0	0	0	1	1	1463.9	1173.48	915.62	355	3908.008
2	0	0	1	0	1	1463.9	1173.48	513.55	92.587	3243.522
3	0	0	1	1	2	1463.9	1173.48	513.55	37.338	3188.273
4	0	1	0	0	1	1463.9	704.351	183.98	92.587	2444.815
5	0	1	0	1	2	1463.9	704.351	183.98	37.338	2389.566
6	0	1	1	0	2	1463.9	704.351	62.303	52.676	2283.231
7	0	1	1	1	3	1463.9	704.351	62.303	277.83	2508.383
8	1	0	0	0	1	927.582	307.005	183.98	92.587	1511.15
9	1	0	0	1	2	927.582	307.005	183.98	37.338	1455.901
10	1	0	1	0	2	927.582	307.005	62.303	52.676	1349.566
11	1	0	1	1	3	927.582	307.005	62.303	277.83	1574.718
12	**1**	**1**	**0**	**0**	**2**	**927.582**	**118.784**	**11.779**	**52.676**	**1110.821**
13	1	1	0	1	3	927.582	118.784	11.779	277.83	1335.973
14	1	1	1	0	3	927.582	118.784	171.02	537.5	1754.883
15	1	1	1	1	4	927.582	118.784	171.02	1079.6	2296.952
										Min J =1110.821

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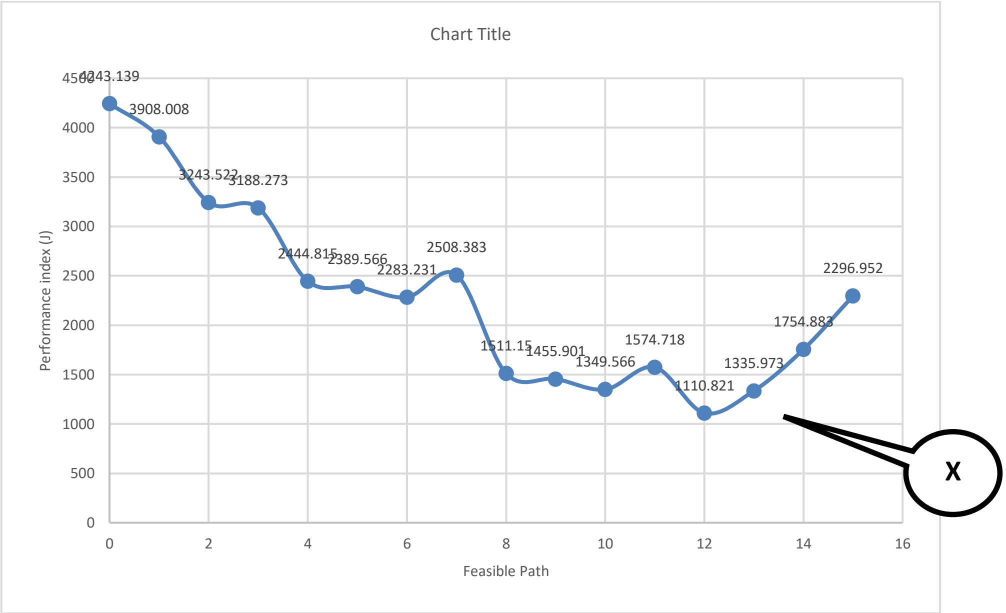

FIGURE 5. Performance Index Against Feasible Path

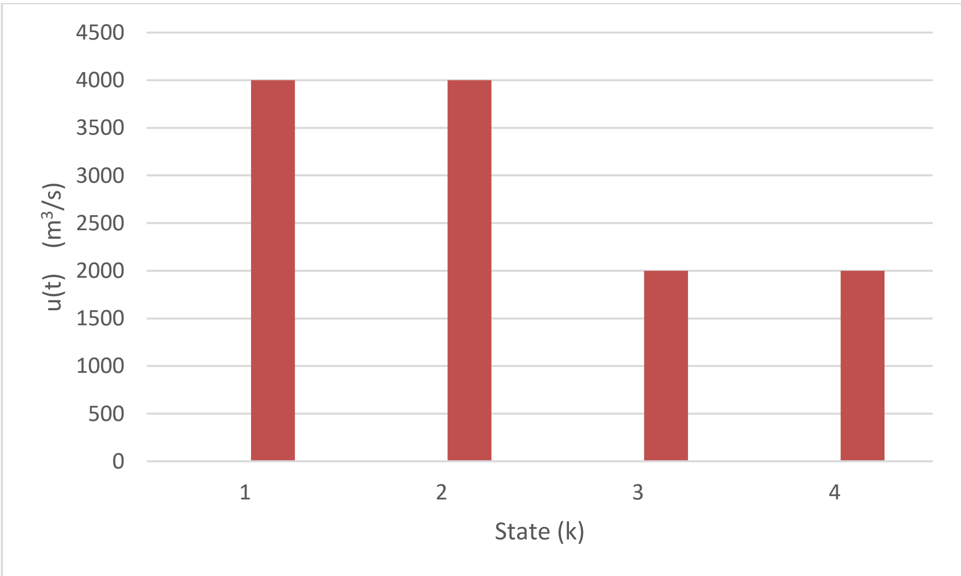

FIGURE 6. Optimal Control

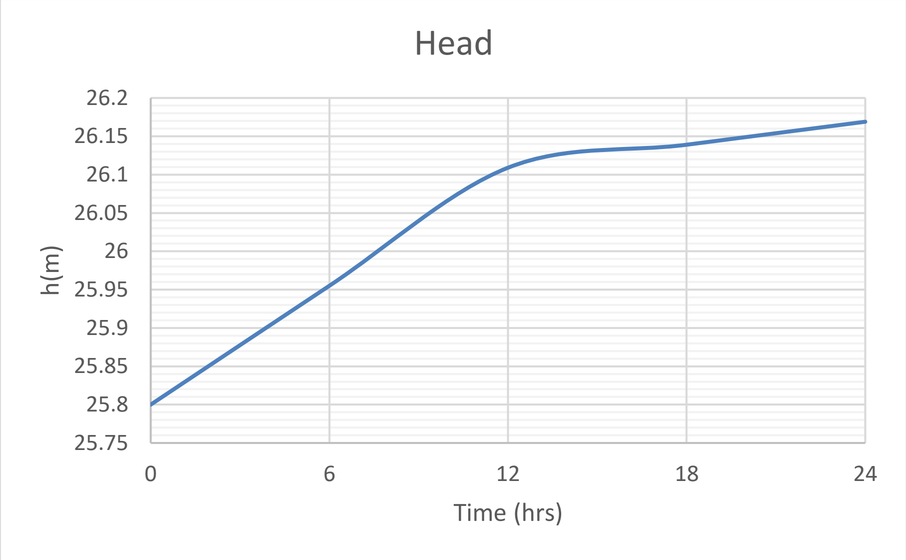

FIGURE 7. Head Trajectory resulting from the Optimal Control

5. CONCLUSION

The paper has presented the potential of the dynamic programming for the solution of optimal control of JHEPS. The algorithm is effective with even two-level of controls. The accuracy can be improved by improving the control level to four but the algorithm becomes computationally intensive and consumes memory space due to the curse of dimensionality. Hence it would be appropriate to maintain to the two levels of controls. Therefore, operators can depend on this control procedure for the optimal management of the scarce generating resources while further work can be carried out for the realization of the physical control system.

REFERENCES

[1] C. T. Thomas, O. Ogunbiyi, M. F. Akorede, K. O. Alabi, B. Yahaya, and B. J. Olufeagba, “Evaluation of Failure and Repair of the Jebba and the Shiroro Hydroelectric Power Stations,” *Springer J. Fail. Anal. Prev.*, vol. 19, no. 2, pp. 488–495, 2019.

[2] A. S. Sambo, B. Garba, I. H. Zarma, and M. M. Gaji, “Electricity Generation and the Present Challenges in the Nigerian Power Sector.,” *J. Energy Power Eng.*, vol. 6, no. 7, pp. 1–17, 2012.

[3] O. D. Jimoh, “Optimized Operation of Kainji Reservoir,” *AU J. Technol.*, vol. 12, no. 1, pp. 34–42, 2008.

[4] T. S. Abdulkadir, A. W. Salami, A. R. Anwar, and A. G. Kareem, “Modelling of Hydropower Reservoir Variables for Energy Generation: Neural Network Approach,” *Ethiop. J. Environ. Stud. Manag. Vol.*, vol. 6, no. 3, pp. 310–316, 2013.

ISSN: 2252-4274 (Print)
ISSN: 2252-5459 (Online)

[5] I. A. O. Omeiza, H. A. Ojongbede, C. T. Thomas, and B. J. Olufeagba, “Assessment of Performance of Turbo-Alternators at The Jebba Hydroelectric Power Station in Nigeria from 2005 – 2014,” *Niger. J. Technol.*, vol. 38, no. 1, pp. 134–141, 2019.

[6] M. S. Zambelli, S. Soares, and D. Da Silva, “Deterministic versus stochastic dynamic programming for long term hydropower scheduling,” *2011 IEEE PES Trondheim PowerTech Power Technol. a Sustain. Soc. POWERTECH 2011*, pp. 1–7, 2011.

[7] A. Husain, “An Overview of Reservoir Systems Operation Techniques,” vol. 4, no. 10, pp. 30–37, 2012.

[8] J. Jacobs *et al.*, “SOCRATES: A system for scheduling hydroelectric generation under uncertainty,” *Ann. Oper. Res.*, vol. 59, no. 1, pp. 99–133, 1995.

[9] O. Ogunbiyi, C. T. Thomas, I. A. O. Omeiza, J. Akanni, and B. J. Olufeagba, “Dynamical Control Model of the Cascaded Kainji-Jebba Hydropower Operating Head,” *FUOYE J. Eng. Technol.*, vol. 4, no. 1, 2019.

[10] H. S. Rodrigues, M. T. T. Monteiro, and D. F. M. Torres, “Optimal Control and Numerical Software: An Overview,” *Syst. Theory Perspect. Appl. Dev.*, pp. 93–110, 2014.

[11] O. Von Stryk, “Numerical Solution of Optimal Control Problems by Direct Collocation,” *Optim. Control - Calc. Var. Optim. Control Theory Numer. Methods, Int. Ser. Numer. Mathmatics*, vol. 111, pp. 129–143, 1993.

[12] S. S. Rao, *Engineering Optimization: Theory and Practice*. 2009.

[13] D. E. Kirk, “Optimal Control Theory: An Introduction.” Dover Publication, New York, 1998.

[14] H. P. Geering, *Optimal Control with Engineering Applications*. Berlin: Springer-Verlag Berlin Heidelberg, 2007.

[15] D. P. Bertsekas, *Dynamic Programming and Optimal Control*, vol. I. 2005.

[16] J. T. Betts, *Practical Methods for Optimal Control Using Nonlinear Programming*. Society for Industrial and Applied Mathematics, 2001.

[17] O. Sundström, D. Ambühl, and L. Guzzella, “On Implementation of Dynamic Programming for Optimal Control Problems with Final State Constraints,” *Oil Gas Sci. Technol. -REv. IFP*, vol. 65, no. 1, pp. 91–102, 2010.

[18] X. Xu and N. Dame, “A dynamic programming approach for optimal control of switched systems - Decision and Control, 2000. Proceedings of the 39th IEEE Conference on,” in *Proceedings of the 39" IEEE Conference on Decision and Control*, 2000, pp. 1822–1827.

[19] X. Wang, H. He, F. Sun, and J. Zhang, “Application Study on the Dynamic

Programming Algorithm for Energy Management of Plug-in Hybrid Electric Vehicles," pp. 3225–3244, 2015.

[20] Z. Stojanovi, D. Vukosavi, D. Divac, N. Milivojevi, and D. Vu, "Hydropower Plants Cascade – Modeling of Short and Long-Term Management," vol. 3, no. 1, pp. 210–227, 2009.

ISSN: 2252-4274 (Print)
ISSN: 2252-5459 (Online)