A Stochastic Modelling Approach to Student Performance Prediction on an Internet-Mediated Environment

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ABSTRACT

Student performance prediction presents institutions and learners with results that assist them to gauge their academic abilities within their context of learning. Performance prediction has been done using different approaches over the years. In this case, stochastic modelling is used and it takes into consideration the use of random variables in the prediction process. The random variables are generated from different scenarios in order to generate a possible output. As a result, the generated output is used to indicate the likelihood of very rare occurrence scenarios which may or may not take place at a future date. With the vast availability of educational data that is available within the learning sector, this data forms the basis of input data that is required for the prediction of student performance within internet-worked environments. This paper develops the prediction model using Stochastic Differential Equations (SDEs). This then gives way to the analysis of data collected from varied respondents within universities leading to the generation of a student performance trajectory.

Keywords: Student Performance, Stochastic Differential Equations (SDEs), Internet Technology, Predictive Model

1. INTRODUCTION

Internet technology refers to a universal conglomeration of interconnected telecommunication networks and channels which facilitate the interaction and interoperability of connected information systems and their users [1]. This technology has brought a revolution in all aspects of world economies by significantly changing the traditional way in which things were done. Although the internet’s growth and development was expected, its immense surge and corresponding impact on different sectors was clearly underestimated. Being the fastest growing technology in the 21st century, the internet has achieved within four years what television revolution achieved in thirteen years. Originally, it was intended for communication within the United States Department of Defense, in case of a nuclear war [2]. This initial intention changed due to the fact that many stakeholders and investors developed great interest in the technology and expanded the scope of its usage.

Specifically, the business sector, the government sector and the education sector have benefited enormously from the advancements that have taken place on the internet. With its great evolution speed, the field of education has not been left behind either. The revolution has created opportunities for students to be active...
learners, allowing instructors to be enablers in the learning process and hence contributing positively to student performance [3, 4]. In the recent past, there has been growing interest in the usage of internet technology in higher learning institutions. This is attributable to the exponential development on new and cheaper technologies that support internet technology. Many technologies and applications that support learning through internet technologies have also promoted the growth of internet technology adoption in learning. This is seen in the use of the massive open online courses (MOOCs) and the use of e-learning platforms which have greatly assisted in availing educational content to students across the globe. This educational content is developed by instructors with a wide range of experience in their subject areas and is therefore considered beneficial to the learners [5, 6].

Many learning institutions in the world particularly in developing countries have embraced internet technology in their institutions with vigor and the belief that it will help enhance scholarly activities in their institution. As such, over the years, policies have been formulated to support the integration of the internet technology in these institutions with expectations of improvement on student performance. Chiefly, large sums of budgetary allocations have been made to support the deployment of internet technology in these institutions. In general, all the shifts taking place in the learning ecosystems in universities in particular, is presumed to lead to enhanced student performance. Consequently, adoption of internet mediated platforms in teaching and learning has become a standard norm in most learning institutions. The fundamental problem addressed in this paper could be if indeed the configuration, adoption and utilization of internet technology in higher learning institution has led to corresponding improved student performance.

This paper investigated the learning process mainly with regard to the utilization of internet technology in the learning process and how it influences student performance in higher learning institutions. Particular focus was on the use of the stochastic modelling approach in the prediction of the student performance trajectory considering different parameters in the learning environment.

2. METHOD: STOCHASTIC PREDICTION MODELLING

By definition, a stochastic process is a family of random variables \( \{ X_\theta \} \) indexed by a parameter \( \theta \), where \( \theta \) belongs to an index set \( \Theta \). When \( \Theta \) represents a set of values of integer type, signifying specific time points, there is a possibility of a stochastic process existing in discrete time or in continuous time. A stochastic process in discrete time occurs when the index set \( \Theta \) is a set of integers representing specific time points. If the index set \( \Theta \) is a set of integers occurring on the real line or within some intervals on the real line, then the stochastic process occurs in continuous time [7].

From a general perspective, a discrete time process with a random variable \( X_n \) will depend on earlier pre-existing values of the process, \( X_{n-1}, X_{n-2}, \ldots \). Likewise, a continuous time process with a random variable \( X(t) \) will equally depend on the values \( X(u) \) for \( u < t \). Therefore, of major interest are conditional distributions which are represented to satisfy the Markov property in the form:

\[
\Pr(X_{t_k} | X_{t_{k-1}}, X_{t_{k-2}}, \ldots, X_{t_1})
\] (1)
for some time $t_k > t_{k-1} > \cdots > t_1$, which depends on the values of $X_{t_k-1}, X_{t_k-2}, \ldots, X_{t_1}$. The Markov property, named after Andrei Markov (1856-1922), states that given the present $(X_{t_k-1})$, the future $X_k$ is independent of the past $X_{t_k-2}, X_{t_k-3}, \ldots, X_1$. The Markov property in (1) can also be rewritten as

$$\Pr(X_{t_k} \mid X_{t_k-1}, X_{t_k-2}, \ldots, X_{t_1}) = \Pr(X_{t_k} \mid X_{t_k-1})$$ (2)

It is referred to as the lack of memory property.

3. STUDENT PERFORMANCE PREDICTION MODEL

Internet Technology usage by student is dependent on their behavioral intentions as explained by the Technology Acceptance Model (TAM). According to the Theory of Reasoned Action (TRA), the behavioral intention of a student like any other technology user is influenced by the students’ attitude and behavior [8, 9]. Therefore, a student will behave (use the internet) based on their attitude and their intentions on the activity to be conducted using the internet.

Research has shown that student performance has been analyzed using many methods. Some of these methods include Data mining techniques [10], Decision tree technique [11] and Factorization Techniques [12]. A new domain of knowledge in predicting student performance is developing around the use of stochastic models in the prediction of student performance. This approach requires the formulation of Stochastic Differential Equations (SDEs) in the form of system equations, observation equations and output equations. The equations are then used to define the model which is validated using a continuous time stochastic modelling R package (CTSM-R) version 3.4.3.

The CTSM-R model structure assumes the nature of object-oriented programming. In this case the mathematical equations are added one at a time. The process begins by initializing the model where an empty model is created and the corresponding mathematical models are added to the object. This is done as,

$$model2 <- ctsm()$$ (3)

where model2 is the model name and ctsm () is the generator function which defines the reference library needed for computations in the model. The object returned is an instance of the class ctsm and the methods help in defining the model structure and the parameter limits.

CTSM-R requires a number of equations to be added in order to assist in generating the desired output. These equations are mainly the system equations, observation equations, the inputs, variance and parameters. In order to add a system equation, the continuous time stochastic differential equations are added into the model by defining them as,

$$model2$addSystem(formula)$$ (4)

the formula is written as a SDE which is valid in R. For instance:
$dP \sim (a \times T^{-a}) \times (Es \times Ls)^{1-a} \times dt + Tc^a \times \Sigma p \times d\omega$

(5)

While adding an observation equation, the equation used takes the form:

\[
\text{model2$addObs(formula)}
\]

(6)

the formula becomes a measurement equation which can take the form:

\[
Y \sim P(h, x, t)
\]

(7)

Another important component that needs to be defined is the variance. The variance of the measurement noise is added for the output with the function,

\[
\text{model2$setVariance(Y~s)}
\]

(8)

Equally important are the input values that will be used in the model. The inputs are the values computed from the collected data and are available for use in the model already defined. This is done by defining the inputs that will be used in the function using the below structure,

\[
\text{model2$addInput(symbol)}
\]

(9)

where the symbol defines which variables are external inputs to the system. For example,

\[
\text{model2$addInput(Tc, Es, Ls)}
\]

(10)

where $Tc$, $Es$, and $Ls$ are all inputs which must be columns in the collected data.

As the model equations are being defined, it is important to consider that there will be values that will be used in the equations and these value could be constants, or values that assume a certain range. Therefore, parameter estimation in CTSM-R is considered by maximizing the likelihood function with an optimization scheme. The scheme is defined by giving the lower limit, the upper limit and the initial values for the parameters. For instance, to define the limits of a variable, the equation is set as,

\[
\text{model2$setParameter(a = c(init = 0.45, lower = 0.1, upper = 0.77))}
\]

(11)

In order to define a specific constant value of a parameter, the function is defined without the limits, for instance,

\[
\text{model2$setParameter(a = c(init = 0.45))}
\]

(12)

With the above in mind, SDEs are formulated to represent an individual student whose actions possess the features of a stochastic process that changes with probability. Student performance within a university is influenced over time $t \geq 0$ by a number of factors which can be grouped as endogenous and exogenous factors. In this paper, consider the technological investment costs ($TI(t)$) at a specific instance in time $t$ as exogenous factors. Student effort ($SE(t)$) in the utilization of
internet resources at a specific instance in time \( t \) and the effectiveness \( (ESE(t)) \) of the student effort in the utilization of internet technology in the learning process at a specific instance in time \( t \) to be the endogenous factors.

3.1 TECHNOLOGICAL INVESTMENT \((TI(t))\)

Technological investment in the context of this paper is framed around the concept of behavioral costs. Behavioral costs capture two important elements that can be used to measure individual student investment in a learning process. These elements include behavioral resources (also referred to as sacrifice) and opportunity costs. For a student to behave (act) in a particular way, he/she must make use of behavioral resources. These relative costs include time costs (time budget), psychic costs (mental budget) and physical costs (physical budget) [13].

In this paper, a fourth cost is introduced, which is identified as the technology costs. This cost touches on aspects to do with technology proficiency or technical attributes of the technology itself, inherent on the student environment and the institutional technology utilization policies.

Hence, the measure of technological investment can be defined as,

\[
TI(t) = \sum f(t_{dc}, Md_{c}, Pb_{c}, T_{c})
\]

where \( t_{dc} \) = time costs as measured on time-demand by an activity, \( Md_{c} \) = Psychic costs as measured on the perceived mental demands of the activity, \( Pb_{c} \) = physical costs as expressed by the physical budget needed for the activity and \( T_{c} \) = the technology costs as expressed by the technological needs/resources/policies for the activity to be done. All these costs are relative except the time costs [14].

Due to the likelihood of individual students borrowing a resource from one another and even seeking assistance from another, the individual costs tend to be shared out. Moreover, some costs are generally shared, for instance, the physical costs. Thus (13) can be written as,

\[
TI(t) = \sum f(t_{dc}, Md_{c}, Pb_{c}, T_{c}) = \frac{\text{Action price}}{\text{Action budget}}
\]

where \( t_{dc} \) = budgeted time costs to undertake an activity, \( Md_{c} \) = Perceived mental demand/psychic costs to completion of an activity, \( Pb_{b} \) = Budgeted resources to undertake an activity and \( T_{c} \) = the budgeted/planned technology costs as expressed by the technological needs/resources/policies completion of an activity.
3.2 STUDENT EFFORT \((SE(t))\)

In this paper, a model that measures the student effort in the utilization of internet technology is presented by considering the behavioral intentions and the student actions [15]. Let the student behavioral intention be denoted by \(BN(t)\) and the action that follows the intention be \(AC(t)\). Then, relating the two attributes gives:

\[
BN(t) = SE(t)(AC(t)) \quad t \geq 0
\]  

(15)

where \(SE_t\) is the relative effort put by the student to carry out a specific action. Equation (15) can be written as,

\[
SE_t = \frac{BN(t)}{AC(t)}
\]  

(16)

with, \(AC(t) = \sum_{t=1}^{m}(sb_t * eo_t)\)

where \(sb_t\) denotes the student belief in carrying out the activity, \(eo_t\) the expected outcome from the activity. Therefore, (16) becomes:

\[
SE_t = \frac{BN(t)}{\sum_{t=1}^{m}(sb_t * eo_t)}
\]  

(17)

Equation (17) gives a relative measure of the effort used by the student to achieve a particular behavioral outcome considering the actions taken at time \(t\). It is assumed that the student behavioural intention will shift depending on a number of other controlling factors.

3.3 EFFECTIVENESS OF STUDENT EFFORT \((ESE(t))\)

In this paper, the effectiveness of student effort in the utilization of internet technology in the learning process will measure the difference between two possible action strategies by the student. More often, students will use preferences in selecting their actions. Effectiveness of student effort in this paper is measured by:

\[
ESE = C_1 - C_2 = w_1 (EO_1 - EO_2) - w_2 (BS_1 - BS_2)
\]  

(18)

where \(C_1 - C_2\) is the relative student preference of action 1 to action 2 which measures the effectiveness of the actions selected. \(EO_1, EO_2\) represent the expected outcome from action 1 and 2 respectively. \(BS_1, BS_2\) represent the behavioural costs associated with action 1 and 2 respectively.
3.4 STUDENT PERFORMANCE ($Pe(t)$)

Assuming the performance of student to be a continuous stochastic process, it can therefore be represented as a nonlinear stochastic system given by:

$$d(Pe(t)) = f((Pe(t), i(t))dt + \sigma(t)d\omega(t), Pe(0) = Pe_0$$

(19)

where the performance state $Pe \in \mathbb{R}^m$, the input $i \in \mathbb{R}^m$, therefore drift term $f: \mathbb{R}^m \times \mathbb{R}^n \rightarrow \mathbb{R}^m$ and the diffusion $\sigma: \mathbb{R}^n \rightarrow \mathbb{R}^m$. The Weiner process $\omega$ in the system equation will model the randomness of the student performance due to unknown errors. Let the student performance outcome be denoted by:

$$Y(t) = f(TI(t), SE, ESE(t)), Y_t \geq 0$$

(20)

Taking student performance at a specific instance in time $t$ to be denoted by $Pe_t$; using the production function of the Cobb-Douglas type, student performance outcome can be modelled as in (21):

$$Pe_t(t, Y(t)) = TIP(SE_t, ESE_t)^q$$

(21)

for some arbitrary but fixed student investment costs, $p \epsilon (0,1)$ and $q = 1 - p$.

The student effort $SE_t$ in the utilization of the internet technology is characterized by random dynamics due to the levels of competences on internet technology and the total productivity factor. It can therefore change over time following a geometric form of the Brownian motion represented by the stochastic differential equation (SDE) as;

$$dSE_t = \mu_{SE}SE_t dt + \sigma_{SE}SE_t dw^{SE}_t, SE_0 > 0$$

(22)

given the average improvement in performance $\mu_{SE} \geq 0$ and constant unpredictable change in technological innovation $\sigma_{SE} > 0, \mu_{SE}SE_t dt$ term represents the drift part and $\sigma_{SE}SE_t dw^{SE}_t$ represents the diffusion term with $dw^{SE}_t$ denoting the a Weiner process which has Gaussian distribution characteristics.

The measure for the effectiveness of student effort in the utilization of internet technology in the learning process can also be captured as geometric Brownian motion given by:

$$dESE_t = \mu_{ESE}ESE_t dt + \sigma_{ESE}ESE_t dw^{ESE}_t, ESE_0 > 0$$

(23)

for an average rate of change $\mu_{ESE} \epsilon \mathbb{R}$ and constant unpredictable change in the effectiveness of student effort, $\sigma_{ESE} > 0$.

Further, it was considered that the technological investment by an institution is directly affected by the student demand for the technology ($S\rho$) and the perceived
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student performance \((PSP)\) due to the utilization of the technology. Demand for internet technology in learning environment can be inferred by the effort put by the student in its usage. Hence, universities are more likely to make policies on investment \((PoI)\) on the internet technology considering the effort put by the students \((SE)\) to use the internet and the expected performance \((PSP)\). These dynamics are captured as:

\[
dTI_t = [Pe_t - SE_t Pol_t]dt + \sigma_{TI} dT_IdW^T, \quad T_I > 0 \tag{24}
\]

where \(\sigma_{TI}\) denotes the constant unpredictability investment, \(Pol_t\) is the random institutional utilization policies rate on internet technology (e.g. broadband sharing ratios, domain separation, platform accessibilities etc.) at time \(t\) and \(\sigma_{TI} > 0\). Therefore, without loss of generality, \(Pol_t\) is dependent on \((Tl_m)_m \geq 0, (SE_m)_m \geq 0, (ESE_m)_m \geq 0\) only and follows a Markovian property of memoryless time-homogeneous.

Hence \(Pol_t = c(Tl_t, SE_t, ESE_t) \forall \text{ institutional utilization policies} \), thus, (24) can be written as:

\[
dTI_t = [Pe_t - SE_t Pol_t(Y_t)]dt + \sigma_{TI} dT_IdW^T, \quad T_I > 0 \tag{25}
\]

From the above equations, the values of \(TI_t, SE_t, ESE_t\) are all influenced by inherent random errors captured as uncertainties which are modelled as independent standard Brownian motions \(dW^T_t, dW^SE_t\) and \(dW^{ESE}_t\) respectively. Equation (25) can be rewritten as,

\[
dTI_t = [TI_t^p(SE_tESE_t)^q - SE_t Pol_t(Y_t)]dt + \sigma_{TI} dT_IdW^T, \quad T_I > 0 \tag{26}
\]

Assuming a student as a consumer of the internet technology is supposed to have a constant rate \(\varphi \geq 0\) of the time-preference and Constant Relative Risk Aversion (CRRA) utility function given by:

\[
u(c_t) = \frac{c_t^{1-\theta} - 1}{1-\theta}\tag{27}
\]

By considering work by [16], assume \(\theta = a\) under circumstances where there is uniformity in the applicability of intervening factors like resources accessibility policies.

Assume internet technology utilization by students in a university is dependent on a collection of institutional policies and internet consumption strategies which was denoted by \(Pol(t,y)\) with \(t\) representing time point and \(y\) is the observable value of \(Y_t\). The desire many learning institutions is to optimize consumption rate of the internet technology as a resource by its student. They thus set utilization strategies and policies as estimated by the utility function given by (27) aimed at aiding students to achieve best performance outcomes. A study by [29] estimates the optimal consumption utility value by
\[ Poi(Y_t) = k \frac{T_I t}{SE_t} \quad k \geq 0 \]  

and by the principle of marginal rate of technical substitution (MRTS) \( k = \frac{b}{a} \). It can be noticed that the optimal consumption rate of the internet technology is dependent on investment costs and the effort. Substituting (28) in (26) gives:

\[ dT_I t = \left[ T_I t^p (SE_t ESE_t)^q - SpT_I t - SE_t k T_I t \right] dt + \sigma_{TI} T_I t \, dw_{TI}, \quad TI_0 > 0 \]  

which simplifies to:

\[ dT_I t = \left[ T_I t^p (SE_t ESE_t)^q - (Sp + k)T_I t \right] dt + \sigma_{TI} T_I t \, dw_{TI}, \quad TI_0 > 0 \]  

Using a similar approach to model student performance, gives:

\[ dPe_t = \mu_{Pe} Pe_t dt + \sigma_{Pe} Pe_t \, dw_{Pe}, \quad Pe_0 > 0 \]  

Considering (21) and applying Ito’s formula on (31), gives:

\[ dPe_t = \mu_{Pe} (TI_t)^p (SE_t ESE_t)^q dt + \sigma_{Pe} (TI_t)^p (SE_t ESE_t)^q \, dw_{Pe}, \quad Pe_0 > 0 \]  

which is the stochastic differential equation (SDE) with initial condition \( Pe_0 = x \), that gives the performance of a student at time \( t \) as shown in (33).

4. METHODOLOGY

Data was collected by the use of questionnaire surveys. The questionnaire had a brief cover letter that introduced the respondents to the study. Precisely, it checked for background information, knowledge of internet usage, capability of the student, attitude of the student, physical environment, the influence to use the technology, utility of the technology, relevance of internet, and the extent of the usefulness of internet. The likert scale used in the questionnaire was a five-point scale ranging from strongly agree to strongly disagree.

The respondents in this case were students from 12 public and 8 private universities who were in their third year of study in a STEM or a non-STEM related course. The students did not need to have used internet technology in their learning process for them to respond to the questionnaire. The sample size given by Slovin’s formula assuming a degree of variability of 0.01 and a confidence level of 95% gave a sample size of 1,000 students.
where \( n \) represents the sample size, \( N \) represents entire population, \( e \) represents the level of precision. Out of the 1,000 questionnaires issued, 796 were returned (79.6% response rate) and only 747 of the returned questionnaires were usable.

Since the target population was very large, there was need to use a sampling technique to obtain a smaller set of the population. This led to the use of stratified random sampling. Stratified random sampling is a procedure for selecting a sample that includes identified sub-groups from the population in the proportion that they exist in the population. The sample size is determined by (35):

\[
    n_k = \left( \frac{N_k}{N} \right) \times n
\]  

(35)

Considering the dynamics surrounding the student learning environment to be of continuous nature, the models used are continuous-time stochastic differential equations. Therefore, a Maximum Likelihood Estimation (MLE) method was used to estimate the parameters. Table 1 gives the parameters that have been used recently in some growth studies for a similar set of prediction [17]. The same parameters were also adopted into the model developed.

### Table 1.
SDE Student Performance Predictor Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>0.1-0.77</td>
<td>[18]</td>
</tr>
<tr>
<td>( \sigma_k )</td>
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<td>[19]</td>
</tr>
<tr>
<td>( \sigma_I )</td>
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<td>[20]</td>
</tr>
<tr>
<td>( \sigma_e )</td>
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<td>[21]</td>
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<tr>
<td>( \mu_E )</td>
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<td>[22]</td>
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<tr>
<td>( \mu_e )</td>
<td>0.01-0.02</td>
<td>[21]</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.05-0.08</td>
<td>[21]</td>
</tr>
<tr>
<td>( \theta )</td>
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<td>[23]</td>
</tr>
</tbody>
</table>

### 5. RESULTS AND DISCUSSION

To test the performance of the SPPM, the data obtained from the survey questionnaires was further refined into independent and dependent variables. The independent variables were classified as Perceived usefulness (PU), Perceived Ease of Use (PEOU), Task Technology Fit (TTF), Attitude (IAtt), Subjective Norm (SN), Knowledge of Internet (KoI), Investment (Inv) and Relevance and Ability (R&A). The dependent variable was classified as Performance (Perf).

Since the data in the research was time-dependent and continuous, a continuous time stochastic modelling R package (CTSM-R) version 3.4.3 was used in the model processing. The package was adopted since it has the capability of handling non-linear stochastic processes. In order to fit the model given by equation (23) into the package, there was need to formulate a set of system of equations that formed the performance analysis algorithm in R. These system equations were categorized as
system equations, observation equations and output equations. The student performance trajectory prediction model system equations, observation equations and the output equations used in this paper were adopted from section 4.4.

Figure 1 gives the predicted student performance trajectory after fitting the model and running the formulated performance prediction algorithm in R-Software.

Figure 1 was generated after considering three predictor variables (investment costs, student effort and effectiveness of student effort). In the graph, x-axis represents input variables (effort, effectiveness and investment) observed together at instance in time. It is assumed that the inputs or dependent variables exist jointly and therefore influence each other in a simultaneous manner. The y-axis represents the generic performance trajectory levels generated representing the expected rate of improvement at different input variable combinations. From the graph, increase in the cumulative values of investment costs, student effort and effectiveness of student effort is seen to contribute to a positive change in performance.

6. CONCLUSION

Predicting student performance is important to the learners and education practitioners. This will help them to understand the implications of whatever resources, strategies and policies they employ. This paper assisted in the formulation of the SDE equations which assisted in the formulation of the predictive model algorithm by focusing on student effort, investment costs and effectives of the strategies they employ in achieving good performance.
At the same time, the paper presents an SDE student performance predictive model which considers student performance as a stochastic process characterized by random noises. It therefore provides educational practitioners with a framework of looking at performance as seen in an instance in time rather than as a longitudinal process that spans a long period of time.

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