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Leaders and Followers Algorithm for Balanced Transportation Problem

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ABSTRACT

Leaders and Followers algorithm is a metaheuristic algorithm which uses two sets of solutions and avoid comparison between random exploratory sample solutions and the best solutions. In this paper, it is used to solve the balanced transportation problem. There are some modifications in the proposed algorithm in order to fit the algorithm to the problem. The proposed algorithm is evaluated using 138 problems. The results are better than the results obtained by other algorithm from previous studies. Overall, Leaders and Followers algorithm has no difficulty in finding optimal solution, even in problems that have large dimension, number of supply and number of demands.

Keywords: Leaders and Followers Algorithm, Transportation Problem

1. INTRODUCTION

Transportation Problem (TP) is one of linear programming problems that is utilized in various fields, e.g., engineering [1], scheduling [2], signature matching [3], finance [4], supply chain [5-6], and traffic management [7]. TP is a constrained optimization problem where the cost must be minimized subject to given capacities of supply and demand. Generally, the goal of TP is to minimize the cost of transporting homogenous product from sources (e.g., factories and warehouses) which have different numbers of supply to destinations (e.g., stores and consumers) which also have different numbers of demand [8].

TP is divided into two groups, i.e., balanced and unbalanced. A balanced transportation problem is the problem where the sum of the supplies from all sources equals to the total demands from all destinations [9]. Otherwise, the problem is called unbalanced transportation problem.

There are many methods for solving TP. There are some classic and widely known methods for solving TP, e.g., Northwest Corner Method (NWC), Least Cost Method (LCM), Vogel Approximation Method (VAM), and MODI (Modified Stepping Stone) [10]. MODI (Modified Distribution) method is algorithm that is used to check the optimality of the solutions obtained by NWC, LCM, or VAM [11]. However, these traditional methods require lengthy calculation time for obtaining the optimal solution [12]. Aramuthakannan and Kandasamy [13] then suggests a method as an alternative for MODI in which the number of iterations is minimized.

Since exact method requires expensive computational cost, metaheuristics that require cheaper computation cost [14] than the exact methods may be promising. There are metaheuristics that has been applied to solve TP, e.g., Genetic Algorithm

(GA), Ant Colony Optimization (ACO) [15], Particle Swarm Optimization (PSO) [15], the hybrid algorithm of Particle Swarm Optimization and Genetic Algorithm (PSOGA). In [17], GA is used to solve Linear TP. However, the results show that GA is very slow. Moreover, in [15], ACO is modified and used to solve TP, but it is still computationally expensive because there are parameters that should be chosen previously. In [18], PSOGA can improve optimal solution, but the computation cost may be expensive.

In [19], Leaders and Followers (LaF) Algorithm is shown to find solutions better than PSO and DE in solving unconstrained non-linear optimization problem. In [20], LaF also performs better in solving constrained non-linear optimization problem than Harmony Search, Firefly Algorithm, Cohort Intelligence, Differential Search, and Musical Composition Method.

However, in the field of optimization, there is an impossibility theorem called No Free Lunch Theorem (NFLT). The theorem generally stated that universal optimizers are impossible [21]. In other words, an algorithm may be better in solving a particular optimization problem, but also may be worse in other optimization problem. Therefore, in this study, LaF is evaluated to solve BTP after having some modifications.

2. MATERIALS AND METHODS

2.1 BALANCED TRANSPORTATION PROBLEM (BTP)

The mathematical model of BTP is as the followings.

$$\min Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{1}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \qquad i = 1, 2, \dots, m \tag{2}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \qquad j = 1, 2, \dots, n \tag{3}$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \tag{4}$$

$$x_{ij} \geq 0, \quad i = 1, 2, \dots, m, j = 1, 2, \dots, n \tag{5}$$

where Z is the objective function or transportation cost; x_{ij} is the decision variable that represents the number of unit that is shipped from source i to destination j; c_{ij} is the shipment cost from source i to destination j; a_i is the number of supply from source i; and b_i is the number of demand from source j for each $i = 1, 2, \dots, m$, and $j = 1, 2, \dots, n$.

2.2 LEADERS AND FOLLOWERS ALGORITHM

Leaders and Followers (LaF) algorithm is a metaheuristic algorithm that utilize two populations, i.e., Leaders and Followers. Basically, these populations have same number of elements but have different purposes. Leaders population is for storing some prospective solutions that may be global optimum, whereas Followers is for exploring a new ‘attraction basin’ (a set of solutions that has local optima). In order

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to prevent premature convergence, LaF avoids direct comparison between newly found solutions and the current best solution.

In the beginning, the populations are randomly generated. There is creation process of new solutions that are the mating result of a randomly picked leader (an element of Leaders population) and a randomly picked follower (an element of Followers population). The new solutions named *trial*. These trials then are compared to the followers. If trial is better than the follower, then the trial replaces the follower position in Followers population.

After a number of the trial creation, comparison, and follower replacement, the medians of the cost of Leaders population and Followers population are compared. If the Followers' is better, then the current best solution will be stored in Leaders and the rest elements of L will be selected using binary tournament selection without replacement. These processes are repeated until the optimal solution is found or termination criterion is satisfied.

2.3 LEADERS AND FOLLOWERS ALGORITHM FOR BALANCED TRANSPORTATION PROBLEM

For solving Balanced Transportation Problem, there has to be some adjustment in Leaders and Followers (LaF) algorithm. Firstly, each leader, the element of Leaders population, and follower, the element of Followers population is the solution in the form of matrix $X = (x_{ij})$ where x_{ij} is the decision variable that represents the number of units that is shipped from source i to destination j for each $i = 1, 2, \dots, m$, and $j = 1, 2, \dots, n$. Since the solution is a matrix of dimension $m \times n$, then Leaders and Followers which are the population of solutions must be three dimensional matrices.

Because in the beginning the solutions are generated randomly, the solution may not match with the numbers of supply from each source and demand by each destination. Therefore, there is an adjusting procedure for each generated solution. Firstly, the elements of line 1 to $m - 1$ are randomly generated with a barrier in order to avoid the sum of each line being over the amount of supply and the sum of each column over the amount of demand. Then, for the line m, the elements are the difference of the amount of demand and the sum of each element in the column, as shown in Equation (6). This equation is to ensure that constraint in Equation (3) is satisfied.

$$x_{mj} = \sum_{i=1}^{m} x_{ij} - b_j \tag{6}$$

However, there is still possibility that the constraint in Equation (2) is not met, because the sum of elements in each line may be less than the amount of supply. So, there will be a repairing process. The process starts by detecting which line is more and less than the amount of supply for each line. For example, the sum of elements in line s is more than a_s whereas the sum of elements in line t is less than a_t where $1 \leq s, t \leq m$. Then, a column k $(1 \leq k \leq n)$ is picked randomly. x_{tk} is renewed by Equation (7) and x_{sk} is renewed by Equation (8).

$$x_{tk\_new} = x_{tk} + \min\left(\left|a_s - \sum_{j=1}^{n} x_{sj}\right|, x_{sk}\right) \tag{7}$$

$$x_{sk\_new} = x_{sk} - \min(\left|a_s - \sum_{j=1}^{n} x_{sj}\right|, x_{sk}) \quad (8)$$

The repairing process is repeated until Equation (2) is satisfied.

After the repairing process, the normal process of LaF algorithm is continued which is trial creation process. It is not the typical trial creation process in LaF, since the solutions are three-dimensional matrices. In this study, the process is modified using a process that is similar to uniform crossover in Genetic Algorithm [22]. For each column j, $trial_{ij}$ for each $i = 1, 2, \dots m$ inherit the $follower_{ij}$ for each $i = 1, 2, \dots m$ or $leader_{ij}$ for each $i = 1, 2, \dots m$ that chosen randomly. The results of this process do not always satisfy the constraints in Equation (2). To handle this problem, the repairing process is performed again. After this process, the process of LaF algorithm is as the same as the basic LaF algorithm. There is median comparison between the cost of leaders and followers, the solutions storing in Leader population, binary tournament selection without replacement, and repetition until the optimal solution is found.

2.4 ALGORITHM EVALUATION

The proposed algorithm is evaluated using 138 balanced test problems, i.e., 12 problems from [23] and Problem 1-2, 4-13, 15-23, 26, 28-33, 35-40, 42-62, 64, 66, 68-83, 85-93, 95, 97-101, 103-140 in [24]. Problem 3, 14, 24, 25, 27, 34, 41, 63, 65, 67, 84, 94, 96, and 102 are not used because they are unbalanced problems. For each problem, the proposed algorithm is tested for two times for each problem. After the evaluation, the results are compared with the results using Vogel's Approximation Method (VAM), Revised Distribution Method (RDI) [13], and The Maximum Range Column Method (MRCM) [25].

3. RESULTS AND DISCUSSIONS

Table 1 presents the evaluation results of Problem 1-12 from [23] using the proposed algorithm (LaF) and the results obtained using Vogel's Approximation Method (VAM), Revised Distribution Method (RDI) [13], and New Method (NM) from [23]. The proposed algorithm finds the optimal solutions. LaF is much better than VAM. Also, LaF is better than the others in Problem 3 and 11. The solutions of Problem 3 and 11 are matrix of dimension 5×5 and 4×4, respectively, which are bigger than the solutions of other problems which only have solution matrix of dimension $3 \times 3, 3 \times 4, 3 \times 5,$ and 4×3. In addition, the numbers of supply and demand in Problem 3 are quite large, i.e., $a = (461, 277, 356, 488, 393)$ and $b = (278, 60, 461, 116, 1060)$. This result shows that LaF has no difficulty in solving such problems.

Table 2 shows the evaluation results of 126 problems from [24] using the proposed algorithm (LaF) and The Maximum Range Column Method (MRCM) [25]. The result shows that LaF only obtains worse solutions in Problem 1 and Problem 57 with only small differences than the solutions obtained by MRCM, i.e., 5 and 10, respectively. However, LaF obtains better solutions than MRCM in Problem 74-80, 82-83, 85, 89, 93, 95, 98, 100-101, 103, 131, 137-139. In other problems, both LaF and MRCM obtain the optimal solutions. The differences of solutions by LaF and

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MRCM in Problem 76, 103 and 131 are more than 1000. Also, in Problem 83 and 139, the differences are more than 100.

Problem 1-140 from [24] has various dimensions, i.e., $3 \times 3, 3 \times 4, 3 \times 5, 3 \times 6, 4 \times 3, 4 \times 4, 4 \times 5, 4 \times 6, 5 \times 3,\ 5 \times 4, 5 \times 5, 6 \times 6,$ and 10×10. The solution of Problem 103 is a matrix of dimension 10×10 with numbers of large supply and demands, i.e., $a =$ (500, 300, 700, 250, 750, 700, 500, 100, 150, 150) and $b = (1000, 500, 200, 300, 300, 600, 100, 500, 400, 200)$. Even so, the proposed algorithm does not have any difficulties in finding optimal solution. Therefore, in overall, the proposed algorithm is better than The Maximum Range Column Method in finding the optimal solution in various balanced transportation problems.

TABLE 1.
Evaluation Results of Problem 1-12 from [23] using the Proposed Algorithm (LaF) and Other Algorithms

Problem	LaF	NM [23]	VAM	RDI [13]
1	412	412	476	412
2	743	743	779	743
3	59356	62484	68804	71710
4	80	80	91	83
5	610	610	780	650
6	3460	3460	3520	3460
7	76	76	80	76
8	506	506	542	506
9	200	200	204	267
10	148	148	150	170
11	180	188	224	272
12	172	172	175	178

4. CONCLUSION

From the result and discussion, it can be concluded that the proposed algorithm, Leaders and Followers, is better than Vogel's Approximation Method (VAM), Revised Distribution Method (RDI), and The Maximum Range Column Method (MRCM) in solving the balanced transportation problems. Moreover, Leaders and Followers algorithm has no difficulty in solving the balanced transportation problems with large dimension and numbers of supply and demands.

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TABLE 2.
Evaluation Results of Problem 1-140 from [14] using the Proposed Algorithm (LaF) and The Maximum Range Column Method (MRCM)

Problem	LaF	MRCM	Problem	LaF	MRCM	Problem	LaF	MRCM
1	29	**24**	50	559	559	98	**134**	140
2	35	35	51	2365	2365	99	271	271
4	62	62	52	412	412	100	**350**	380
5	4525	4525	53	960	960	101	**968**	988
6	505	505	54	674	674	103	**62470**	68500
7	2350	2350	55	76	76	104	2424	2424
8	15650	15650	56	172	172	105	3300	3300
9	380	380	57	1690	**1680**	106	980	980
10	1200	1200	58	420	420	107	4450	4450
11	1130	1130	59	5950	5950	108	1140	1140
12	1350	1350	60	184	184	109	920	920
13	14150	14150	61	267	267	110	735	735
15	143	143	62	695	695	111	1620	1620
16	173	173	64	6400	6400	112	68	68
17	743	743	66	3320	3320	113	80	80
18	610	610	68	555	555	114	740	740
19	3460	3460	69	625	625	115	17300	17300
20	506	506	70	590	590	116	4840	4840
21	886	886	71	390	390	117	630	630
22	118	118	72	830	830	118	1160	1160
23	2100	2100	73	47250	47250	119	515	515
26	140	140	74	**2486**	2517	120	13650	13650
28	796	796	75	**410**	415	121	790	790
29	635	635	76	**59356**	60448	122	43476	43476
30	3100	3100	77	**12100**	12200	123	4525	4525
31	56	56	78	**776**	781	124	9200	9200
32	12220	12220	79	**510**	535	125	2750	2750
33	7350	7350	80	**995**	1005	126	1650	1650
35	900	900	81	2882	2550	127	6445	6445
36	920	920	82	**105**	113	128	13695	13695
37	730	730	83	**20550**	20970	129	412	412
38	235	235	85	**148**	150	130	743	743
39	44100	44100	86	25	25	131	**59356**	60448
40	102	102	87	26	26	132	80	80
42	7350	7350	88	134	134	133	610	610
43	3400	3400	89	**102**	112	134	3460	3460
44	772	772	90	332	332	135	76	76
45	2595	2595	91	230	230	136	506	506
46	2221	2221	92	1860	1860	137	**200**	208
47	799	799	93	**50**	56	138	**148**	152
48	47	47	95	**76**	80	139	**180**	327
49	173	173	97	750	750	140	172	172

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